

# Temporal Reasoning Based on Semi-Intervals

Christian Freksa\*  
Institut für Informatik  
Technische Universität München  
Arcisstr. 21  
8000 München 2  
Germany

## Abstract

A generalization of Allen's interval-based approach to temporal reasoning is presented. The notion of 'conceptual neighborhood' of qualitative relations between events is central to the presented approach. Relations between semi-intervals rather than intervals are used as the basic units of knowledge. Semi-intervals correspond to temporal beginnings or endings of events. We demonstrate the advantages of reasoning on the basis of semi-intervals: 1) semi-intervals are rather natural entities both from a cognitive and from a computational point of view; 2) coarse knowledge can be processed directly; computational effort is saved; 3) incomplete knowledge about events can be fully exploited; 4) incomplete inferences made on the basis of complete knowledge can be used directly for further inference steps; 5) there is no trade-off in computational strength for the added flexibility and efficiency; 6) for a natural subset of Allen's algebra, global consistency can be guaranteed in polynomial time; 7) knowledge about relations between events can be represented much more compactly.

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*TIME IS A MASK WORN BY SPACE*

Robert Fulton [6]

## 1 Introduction

### 1.1 Background

In his paper on maintaining knowledge about temporal intervals James Allen introduces a temporal logic based on intervals and their qualitative relationships in time [1]. Allen's approach is simple, transparent, and easy to implement. The basic elements of Allen's theory are intervals corresponding to events (rather than points corresponding to instants), qualitative relations between these intervals, and an algebra for reasoning about relations between intervals.

The appeal of Allen's approach has triggered a variety of research enterprises within and beyond temporal reasoning. For example, Allen and Hayes [2, 10] and Ladkin [14] develop axiomatic frameworks for the theory; Vilain, Kautz, van Beek [20, 21] and Nökel [19] study the computational complexity of Allen's reasoning scheme and of some variants; Gsgen [9], Mukerjee and Joe [18], Freksa [7], and Hernandez [12] transfer the approach to the spatial domain; Ligozat [16] generalizes the interval-concept for reasoning with chains of events; Dean and Boddy [4] and Dubois and Prade [5] focus on incomplete and fuzzy knowledge; Ladkin [15] presents a survey of interval-based constraint reasoning and a selected bibliography.

## 1.2 A cognitive perspective

The present paper approaches the issue of representing time and temporal reasoning from a cognitive perspective: in addition to the logical constraints considered by Allen, we take into account neighborhood relationships between temporal relations; this is motivated primarily by physical constraints on perception. These relationships permit to restrict Allen's algebra in an interesting way. The result is increased inferencing efficiency while full reasoning power is maintained. The inferencing behavior of the modified approach becomes 'cognitively plausible' in several respects. A high degree of regularity in Allen's knowledge base becomes visible through the additional relationships; this allows for a drastic compaction of the inferencing knowledge base.

Allen [1] discusses the formal problem that arises when representing instantaneous events by points on the real line. This problem is due to the fact that logical inconsistencies arise when events are allowed to have zero duration. Besides the arguments Allen provides against the use of points on the real line, namely physical and logical arguments, they are not appropriate for modelling events from a cognitive perspective either. We know that events have to have a certain extent, both in time and in space, in order to be perceivable [11].

Hayes and Allen [10] distinguish between events, which always have some duration, and durationless abstract time points – temporal locations associated with events or with transitions between events. In the present paper we only consider 'real' events as in [1] and we agree with Allen that they must not be represented by points on the real line. We also agree that qualitative knowledge about temporal affairs can be based on events. However, we do not agree with Allen's conclusion that intervals should be used as the representational primitives for reasoning about events.

We must carefully distinguish between an ontological representation of temporal relations, i.e., the representation of a specific set of mutually compatible temporal relations, and

the representation of *knowledge about* temporal relations. If we know everything about all relations, the distinction is insignificant; but if we deal with incomplete knowledge, this makes a big difference. Typically, we do not have complete knowledge about temporal relations between events to start with; but even if we do, after only one inference step we may not have complete knowledge about the inferred relations.

Allen's interval-based approach favors the representation of ontological states of affairs: a completely known temporal relation between two events is expressed by a simple relation between two intervals. The representation of incomplete knowledge, on the other hand, creates a cognitively awkward situation: the less we know, the more complex the representation of what we know becomes. What is known is represented in terms of disjunctions of what *could be* the case.

From a cognitive point of view, we prefer to represent what is known more directly and in such a way that less knowledge corresponds to a simpler representation than more knowledge does. For this reason and for reasons stated in the following sections we will use 'beginnings' and 'endings' of intervals as representational primitives. We may only know the temporal beginning or ending of an event. For example, we may only have information about the birth or the death of a person, but not both; or we may know that a certain event Y did not start before a given event X, but we do not know if X and Y started simultaneously or if Y started after X. In many cases useful inferences can be drawn from such incomplete knowledge, in some cases even without any loss of information.

## 2 Temporal knowledge about the physical world

An event is something that happens. Beginnings of events always take place before their endings. If we let the beginnings and endings of two events have three possible qualitative relations:  $<$ ,  $=$ ,  $>$ , then two events which start in a beginning and terminate in an ending have

thirteen possible qualitative relations [3]. These correspond to the relations that two ordered pairs of real numbers (the boundaries of real-valued intervals) can have under the relations  $<$ ,  $=$ ,  $>$ .

Note that we do not assume that beginnings and endings of events correspond to the end points of real-valued intervals [2]. Rather, beginnings and endings are considered (recursively) as events themselves. Thus, at one level of consideration beginnings and endings of events will appear as atoms (conceptual points) while at a higher resolution they will appear as grains which themselves start in beginnings and terminate in endings.

Allen denotes the thirteen relations between two events with *before* ( $<$ ), *after* ( $>$ ), *during* ( $d$ ), *contains* ( $di$ ), *overlaps* ( $o$ ), *overlapped-by* ( $oi$ ), *meets* ( $m$ ), *met-by* ( $mi$ ), *starts* ( $s$ ), *started-by* ( $si$ ), *finishes* ( $f$ ), *finished-by* ( $fi$ ), and *equals* ( $=$ ). Figure 1 associates the thirteen relations by means of a four-coordinate table with the corresponding relations between the beginnings  $\alpha$ ,  $A$  and the endings  $\omega$ ,  $\Omega$  of the two events. The figure shows how the relations may be distinguished by considering only a subset of relations between beginnings and endings. For example, to distinguish the relation *before* ( $<$ ) from the twelve other relations it is sufficient to note that  $\omega < A$  and to distinguish the relation *starts* ( $s$ ) from the other relations it is sufficient to note that  $\alpha = A$  and  $\omega < \Omega$ . In no case, more than two relations between beginnings and endings of events must be known for uniquely identifying the relation between the corresponding events.

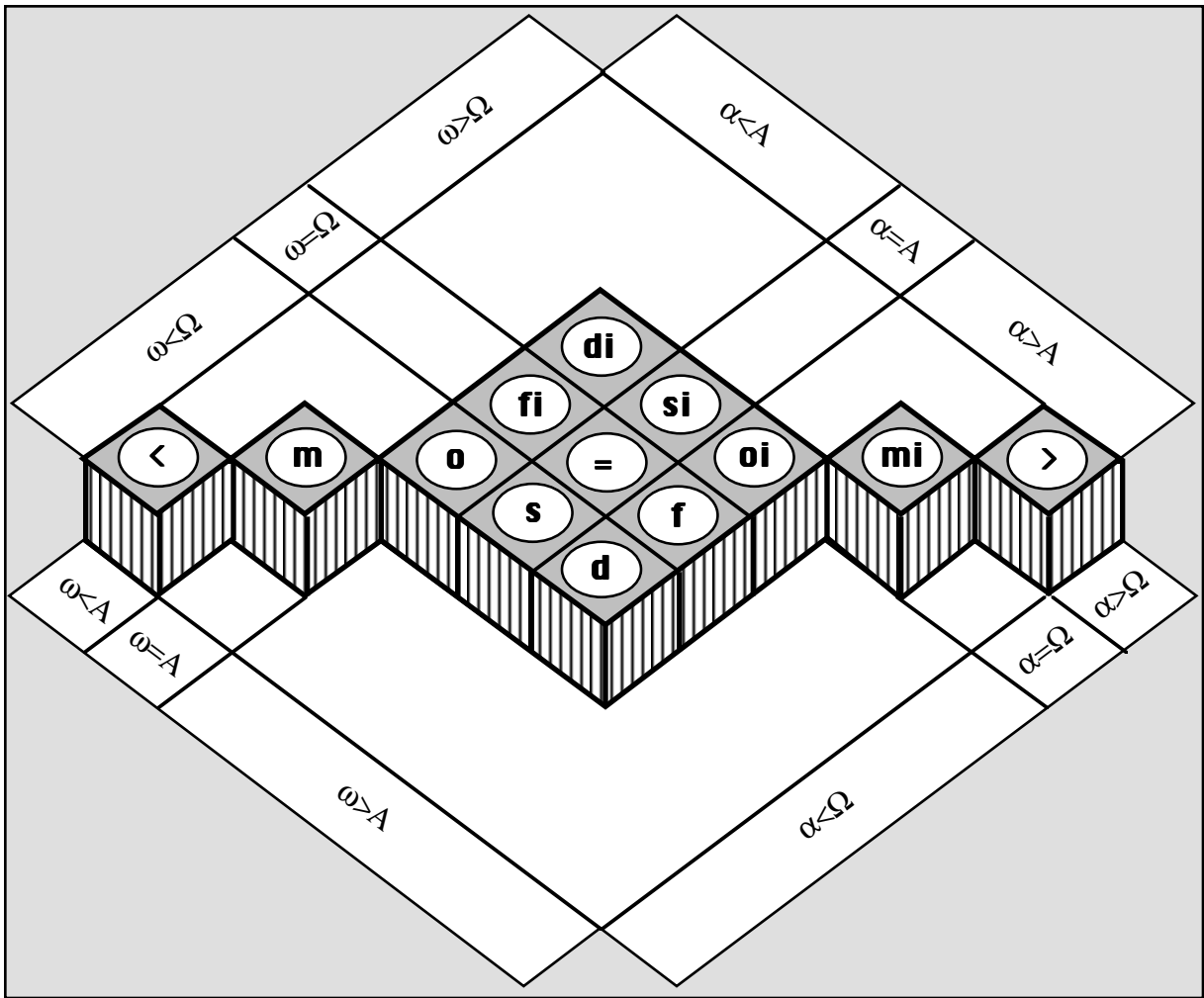


Figure 1: The thirteen qualitative relations between two events characterized by relations between their beginnings  $\alpha$ ,  $A$  and their endings  $\omega$ ,  $\Omega$ .

The reason that such incomplete information about events suffices for fully characterizing their qualitative relations is due to two domain-inherent conditions: 1) the beginnings of events take place before their endings ( $\alpha < \omega$ ,  $A < \Omega$ ) and 2) the relations  $<$ ,  $=$ ,  $>$  are transitive. Without these conditions,  $3^4 = 81$  relations between the four beginnings and endings of two events would be possible.

Allen uses the thirteen possible relations between two events as a basis for a theoretical framework for temporal reasoning. In addition to Allen's theory we will take into account considerations about how cognitive systems establish relations from observing the real world. In observing the real world, there will be situations in which only partial knowledge about the domain is available and in which uncertainty exists as to which of the mutually exclusive abstract relations holds.

### 2.1 Incomplete knowledge about events

In many temporal reasoning situations we do not have to know everything about the involved events in order to infer what we want to know. For example, in order to determine that Newton lived *before* Einstein it is sufficient to know that Newton's death took place before Einstein's birth; it does not help if in addition we know when Newton was born or when Einstein died. Actually, we can derive complete qualitative knowledge about the relations between the birth and death dates involved, due to the domain-inherent conditions mentioned in the previous section.

Of course we may encounter situations in which the available knowledge is insufficient for determining the complete answer to a query; however, a partial answer may be better than no answer at all. For example, we may want to know if two artists may have been influenced by each other. All we know is that X was born before Y's death and that X died after Y. We do not know who was born first. From this information we can conclude that Y lived *during* X' lifetime or he *started* X' lifetime or his life *overlapped* with X' life. Although we can not infer who was the older artist or which was the period when they both lived, at least we know that there was a common period.

With Allen's representation, it is possible to express the situation of the example given above as follows: "X was born before Y's death" can be expressed as "X lived *before* Y or X' life *meets* Y's life or X' life *overlaps* Y's life or X' life is *finished by* Y's life or X' life *contains*

Y's life or X' life is *started by* Y's life or X' life *equals* Y's life or X' life *starts* Y's life or X lived *during* Y's life or X *finishes* Y's life or X' life is *overlapped by* Y's life", and "X died after Y" can be expressed as "X' life *contains* Y's life or X' life is *started by* Y's life or X' life is *overlapped by* Y's life or X' life is *started by* Y's life or X lived *after* Y". The inference step then consists of forming the conjunction of the two sets of disjunctions: "X' life *contains* Y's life or X' life is *started by* Y's life or X' life is *overlapped by* Y's life" which is equivalent to the conclusion derived above.

## 2.2 Neighborhoods vs. disjunctions

As suggested in the introduction, it does not appear cognitively adequate to represent coarse knowledge in terms of disjunctions of finely grained alternative propositions, although this representation may be logically correct. Coarse knowledge is a special form of incomplete knowledge. The missing knowledge corresponds to fine distinctions which are not made. The alternatives all lie in the same ballpark of a conceptualization, they are 'conceptual neighbors'. For use in future parts of this paper, we make the following definitions:

Definition 1: Two relations between pairs of events are (*conceptual*) *neighbors*, if they can be directly transformed into one another by continuously deforming (i.e. shortening, lengthening, moving) the events (in a topological sense).

Examples: The relations *before* ( $<$ ) and *meets* ( $m$ ) are conceptual neighbors, since they can be transformed into one another directly by lengthening one of the events.

The relations *before* ( $<$ ) and *overlaps* ( $o$ ) are not conceptual neighbors, since a transformation by means of continuous deformation only can take place indirectly (via the relation *meets* ( $m$ )).



Definition 2: A set of relations between pairs of events forms a (*conceptual*) *neighborhood* if its elements are path-connected through ‘conceptual neighbor’ relations.

Examples: The relations *before* ( $<$ ), *meets* ( $m$ ), and *overlaps* ( $o$ ) form a conceptual neighborhood since they can be transformed into one another by a chain of direct continuous deformations of the associated events and all three relations are contained in the neighborhood.

The relations *before* ( $<$ ) and *overlaps* ( $o$ ) do not form a conceptual neighborhood.

Note: For reasons of consistency, the two degenerate cases of a single relation and of all thirteen relations are included in this definition.

Definition 3: Incomplete knowledge about relations is called *coarse knowledge* if the corresponding disjunction of at least two relations forms a conceptual neighborhood.

Examples: The disjunction *before or meets or overlaps* ( $< m o$ ) represents coarse knowledge about the relation between two events; the disjunction *before or overlaps* ( $< o$ ) does not represent coarse knowledge.

Note: The case of a single relation is excluded here, since this case corresponds to complete knowledge.

If temporal relations are perceived incompletely, the resulting knowledge is typically coarse knowledge. A perception channel will not generate the set of alternatives  $X m Y$  or  $X o Y$  or  $X o_i Y$  ([9, Figure 3]), for example. The reason that these alternatives are not generated without the intermediate relations is that the last two alternatives of this disjunction

have drastically different perceptual appearances – they vary in several aspects. If the system cannot distinguish alternatives differing in several aspects, then it cannot distinguish alternatives differing only in a subset of these aspects; thus, it will consider the neighboring intermediate alternatives as well. As we will show in chapter 4, temporal reasoning on the basis of the thirteen interval relations will either yield complete knowledge or coarse knowledge, but never scattered disjunctions.

Incomplete knowledge consisting of non-neighboring alternatives may be available, however, from more abstract knowledge sources. For example, a story understanding system may have knowledge about the qualitative relation of two events  $X$  and  $Y$  but lack knowledge about their identity. Thus, two non-neighboring alternatives  $X < Y$  and  $X > Y$  cannot be distinguished while neighboring alternatives can. In this case, two distinct (mental) images would correspond to the two alternatives, rather than a single coarse image. In such situations, it appears ‘cognitively justified’ to use the abstract concept of a disjunction for the representation of alternatives.

Thus, we make a distinction between knowledge incompleteness which does not permit a fine *resolution* of closely related variants within a neighborhood (lack of knowledge about details) and incompleteness which does not permit the *selection* of the appropriate alternative (lack of knowledge about essentials). In the former situation, we want to express knowledge directly on the granularity level on which it is available, i.e., we represent neighborhoods. As a side-effect, we will have to represent only knowledge which is positively available; we do not have to carry along the burden of the possibilities that remain open due to lack of more detailed knowledge. In the latter situation, knowledge representation in terms of disjunctions may be appropriate. However, by restructuring the knowledge, it may be possible to find representations in terms of neighborhoods, for such situations as well.

### 3 Semi-intervals and conceptual neighborhoods

According to the considerations presented in the previous sections, we will represent knowledge about time in terms of relationships between beginnings and endings of events. We call beginnings and endings of events ‘semi-intervals’. (Semi-intervals are equivalent to what Allen and Hayes call ‘nests’ [2]). In relating events, an ending of an event will be called ‘equal’ to the beginning of another event if the former event meets the latter. In order to support the following discussion mnemonically, we will introduce special labels for the relationships between semi-intervals. These labels will be used in addition to the labels introduced by Allen.

We will say *X is older (ol)* than *Y* when the beginning of *X* is less than the beginning of *Y*. *X is head to head (hh)* with *Y* when their beginnings are equal and *X is younger (yo)* than *Y* when the beginning of *X* is greater than the beginning of *Y*. Accordingly, we will say, *X is survived by (sb)* *Y*, *X is tail to tail (tt)* with *Y*, or *X survives (sv)* *Y* when the ending of *X* is less than, equal, or greater than the ending of *Y*, respectively. We will say *X precedes (pr)* *Y*, when the ending of *X* is not greater than the beginning of *Y*, we will say, *X succeeds (sd)* *Y*, when the beginning of *X* is not less than the ending of *Y*, otherwise (i.e., when the intersection of *X* and *Y* is not empty) *X is a contemporary (ct)* of *Y*. If *X* does not precede *Y*, we will say *X is born before death of (bd)* *Y* and if *X* does not succeed *Y*, we will say *X died after birth of (db)* *Y*. These relations are shown in Figure 2 (compare Allen [1983], Figure 2).

Relation	Label	Inverse	Illustration
<i>X is older than Y</i> <i>Y is younger than X</i>	ol	yo	XXX???? YY
<i>X is head to head with Y</i>	hh	hh	XXX?? YYYY
<i>X survives Y</i> <i>Y is survived by X</i>	sv	sb	????XXX YY
<i>X is tail to tail with Y</i>	tt	tt	??XXX YYYY

X precedes Y	pr	XXX?
Y succeeds X	sd	YYY
X is a contemporary of Y	ct	?XXX???
	ct	???YYY?
X is born before death of Y	bd	XXX?????
Y died after birth of X	db	?????YYY

Figure 2: Eleven semi-interval relationships. Question marks (?) in the pictorial illustration stand for either the symbol denoting the event depicted in the same line (X or Y) or for a blank. The number of question marks reflects the number of qualitatively alternative implementations of the given relation.

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Combining constraints from above we obtain the relations *older & survived by* (ob), *younger & survives* (ys), *older contemporary* (oc), *surviving contemporary* (sc), *survived-by contemporary* (bc), and *younger contemporary* (yc).

### 3.1 Uncertainty about temporal relations

Allen’s composition table [1, Figure 4] establishes the set of theoretically possible relations between two intervals which both have a known qualitative relation to a third interval. The table does not represent knowledge about the effects of small variations or degradations in the input knowledge, specifically, lack of knowledge about certain details. Such variations may be present in the knowledge about the real world due to perceptual uncertainty and/or the dynamics of the domain. For example, we may not know if event X takes place *before* event Y, if X *meets* Y, or if X *overlaps* Y, but we can distinguish these three options from the remaining ten alternatives.

Uncertainty as to which temporal relation holds between two events does not typically mean that any of the thirteen relations are considered possible by a perceiving cognitive system

– otherwise the system is not perceiving. Rather, uncertainty may exist between few options. Furthermore, perceptual uncertainties usually do not cause large jumps in the conclusions; rather, conceptually related options are obtained. In order to model such lateral knowledge dependencies, we structure the temporal relations between events according to a conceptual neighborhood relation. This neighborhood relation is determined by our understanding as to which uncertainties in perception are physically feasible and/or cognitively plausible.

### 3.2 Conceptual neighborhoods among temporal relations

*natura non facit saltus*

Linnæus

According to our definition of conceptual neighborhood in section 2.2, we arrange the thirteen mutually exclusive relations between events in such a way that conceptually neighboring relations become neighbors in our depiction. Figure 3 shows this arrangement. The two events are depicted by a dumbbell-shaped line and a rectangle, respectively; time is assumed to proceed from left to right. We obtain two dimensions of neighborhood: the vertical dimension corresponds to the relative times at which the events take place; the horizontal dimension corresponds to the relative duration of the events.

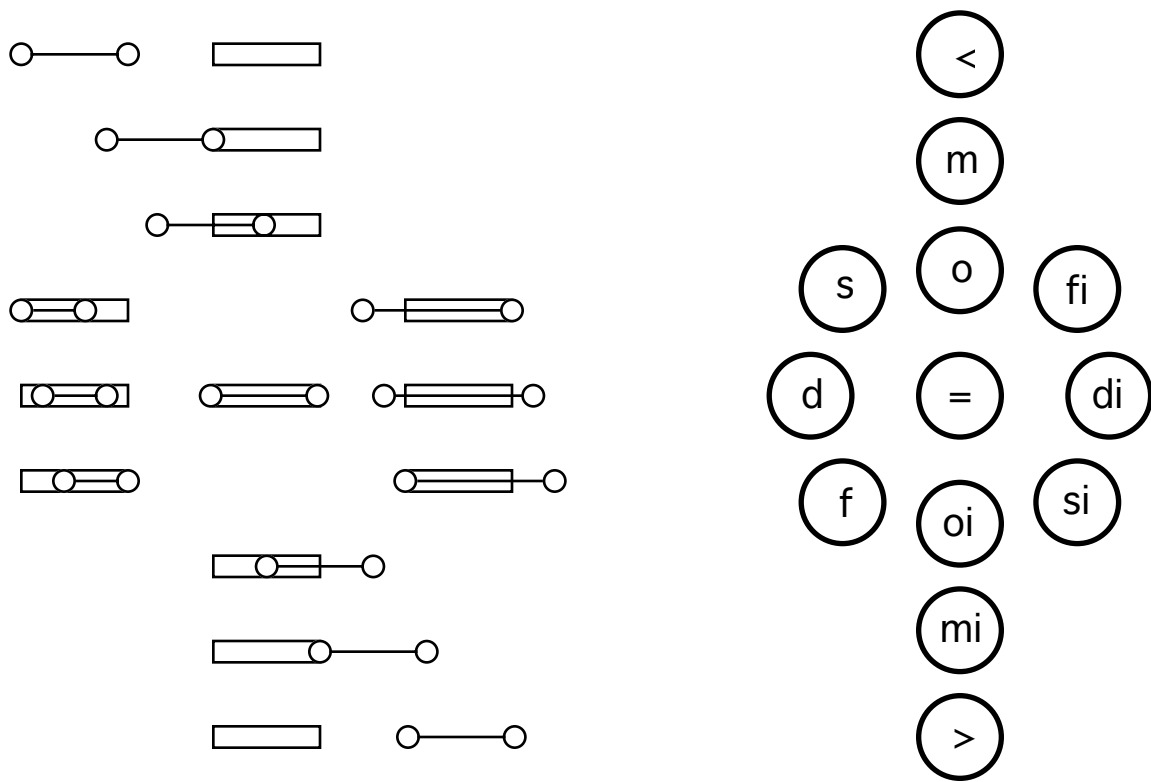


Figure 3: Left: Temporal relations between two events arranged according to their conceptual neighborhood. Right: The corresponding labels arranged accordingly.

Depending on the types of deformation of events and their relations, we obtain different neighborhood structures. If we fix three of the four semi-intervals of two events and allow the fourth to be moved, we obtain the *A-neighbor* relation (Figure 4). If we leave the duration of events fixed and allow complete events to be moved in time, we obtain the *B-neighbor* relation. If we leave the ‘temporal location’ of an event fixed (reflected, for example, by the midpoint of the corresponding interval) and allow the duration of the events to vary, we obtain the *C-neighbor* relation.

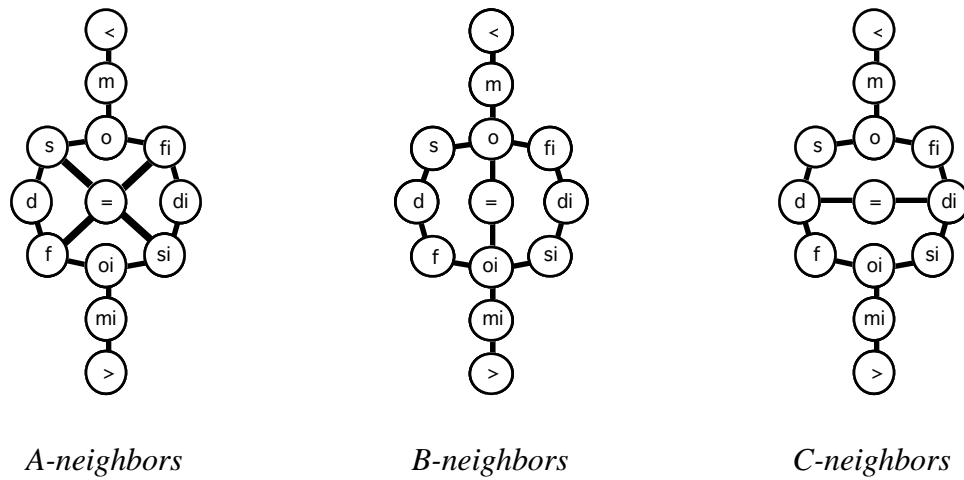


Figure 4: Differing deformations of events induce different neighborhood structures.

Which of these options is most appropriate, may depend on the specific domain of reasoning. Unless otherwise noted, the statements in this paper are independent of the particular choice. We therefore depict the most demanding situation with the most liberal interpretation of the neighborhood relation where all three neighbor relations are permitted.

For easy visual reference to the thirteen temporal relations between two intervals and their neighborhood relations as depicted in Figure 3, we will use icons symbolizing the neighborhood structure as shown in Figure 5. The black dots indicate which of the thirteen relations within the structure is being referred to. Below the icons the corresponding labels are indicated.

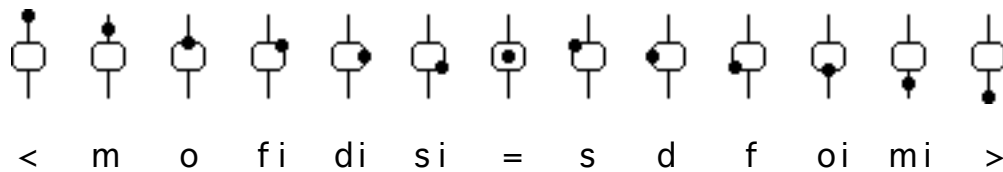


Figure 5: The thirteen qualitative relations between intervals depicted by icons.

I am putting forward the thesis here that if a cognitive system is uncertain as to which relation between two events holds, uncertainty can be expected particularly between neighboring concepts. The introduction of movement into our static domain of relations is not intended to make a complex situation even more difficult; rather I suggest that we easily can discover neighboring concepts by imagining gradual changes in the represented world and by observing the corresponding state transitions in the conceptual world. This will turn out to be very helpful for adequately representing static situations, particularly for representing uncertainty.

## 4 Neighborhood-based reasoning

In this chapter, we will first revisit temporal reasoning on the basis of the thirteen relations used by Allen. Allen's composition table is discussed in the context of the neighborhood-based representation. This leads to some observations regarding the structure of temporal knowledge. On the basis of these observations, conceptual neighborhood is exploited more radically. The resulting approach is presented in the remainder of the chapter.

In order to visualize the use of the conceptual neighborhood relations, we present Allen's composition table in an arrangement which preserves some of the neighborhood relations: the rows and columns are arranged in such a way that neighboring rows and columns always correspond to neighboring preconditions in the sense defined above. The two neighborhood dimensions span a 4-dimensional ( $2 \times 2$ ) composition structure. Since it is not easy to depict a 4-dimensional structure on paper,<sup>1</sup> we depict a linearized version of the structure: in Figure 6 we arrange the rows and columns according to the following sequence: <, m, o, fi, di, si, =,

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<sup>1</sup> For the discovery of the regularities and the development of the reasoning system presented, a *thinktool* was implemented in HyperCard which aided in representing and manipulating the 4-dimensional neighborhood structure.



s, d, f, oi, mi, >. (This is one of two possible ways of listing the relations such that each is listed exactly once and that all neighbors in the list correspond to neighbors in Figure 3.)

In addition, in the new composition table temporal relations are presented in canonical form by icons as developed in the previous section (instead of arbitrarily arranged mnemonic labels). This is done for several reasons: 1) A canonical arrangement of relations makes the regularity of the internal structures more visible; 2) the conceptual neighborhood relations between temporal relations are directly reflected in the icon structure; 3) the icons allow for the direct representation of coarse relations (rather than as disjunctions of fine relations); and 4) the representations can be used directly for performing simple operations. Coarse relations are represented by superposition of the corresponding icons from Figure 5. For example,



corresponds to the disjunction of the relations <, m, o, s, d, and



corresponds to the disjunction of all thirteen relations which means that no constraint on the relationships between the events is given.

<													
m													
o													
fi													
di													
si													
=													
s													
d													
f													
oi													
mi													
>													

Figure 6: Allen's composition table (incl. =) arranged to preserve conceptual neighborhood.

Looking at the composition structure depicted in Figure 6, we can make a number of observations:

- 1) only a fraction of the inferences that can be drawn from the composition table are given in terms of unique relations between intervals; most of the conclusions appear in terms of disjunctions of alternative relations;
- 2) the sets of alternative relations in the entries of Allen's composition table always form conceptual neighborhoods;
- 3) in many cases, the transition to neighboring entries leads to sub-neighborhoods or super-neighborhoods rather than to completely different relations;
- 4) the transition to neighboring entries never causes to a jump to non-neighboring relations;
- 5) only a small fraction of combinatorially possible neighborhoods actually appears in the table;
- 6) there is a lot of symmetry which may be exploited for temporal reasoning;
- 7) the observations are valid not only with respect to the presented linearized neighborhood table; they hold for the complete 4-dimensional structure.

Observations 1) and 2) should not be completely surprising, since the structure exhibits gradual transitions from one qualitative state to another; only one (micro-) feature is changed at a time; these microfeature transitions correspond to the neighborhood relations in the structure. Observation 3) is very useful since it allows for reasoning under uncertainty. Observations 4) and 5) provoke the question of the cognitive significance of the neighborhoods. If the neighborhoods appear to correspond to cognitive relevant concepts, may be the reasoning should be done based on these neighborhoods which correspond to classes of relations rather than on the individual relations themselves.

Note that the neighborhoods that are found in the table either are contained in our list of concepts derived from relating semi-intervals (Figure 2) or are obtained by conjoining such concepts (except for the non-informative entry ? corresponding to the disjunction of all thirteen relations between two intervals). Figure 7 associates the icons and their corresponding neighborhoods with their mnemonics, their associated labels, the corresponding list of Allen-relations and the corresponding constraints between beginnings and endings of the respective events. With help from Figure 7 we can read the composition table as follows: If X *meets* Y and Y is *after* Z then X *survives* Z; or: If X *overlaps* Y and Y is *overlapped by* Z then X is a *contemporary* of Z, etc.

ICON	LABEL	MNEMONIC	ALLEN	CONSTRAINTS
	?		< m o fi di si = s d f oi mi >	none
	ol	<i>older</i>	< m o fi di	$\alpha < A$
	hh	<i>head to head with</i>	si = s	$\alpha = A$
	yo	<i>younger</i>	d f oi mi >	$\alpha > A$
	sb	<i>survived by</i>	< m o s d	$\omega < \Omega$
	tt	<i>tail to tail with</i>	fi = f	$\omega = \Omega$
	sv	<i>survives</i>	di si oi mi >	$\omega > \Omega$
	pr	<i>precedes</i>	< m	$\omega \leq A$
	bd	<i>born before death of</i>	< m o fi di si = s d f oi	$\alpha < \Omega$
	ct	<i>contemporary of</i>	o fi di si = s d f oi	$\alpha < \Omega, \omega > A$
	db	<i>died after birth of</i>	o fi di si = s d f oi mi >	$\omega > A$
	sd	<i>succeeds</i>	mi >	$\alpha \geq \Omega$
	ob	<i>older &amp; survived by</i>	< m o	$\alpha < A, \omega < \Omega$
	oc	<i>older contemporary of</i>	o fi di	$\alpha < A, \omega > A$
	sc	<i>surviving contemporary of</i>	di si oi	$\alpha < \Omega, \omega > \Omega$
	bc	<i>survived by contemporary of</i>	o s d	$\omega > A, \omega < \Omega$
	yc	<i>younger contemporary of</i>	d f oi	$\alpha > A, \alpha < \Omega$
	ys	<i>younger &amp; survives</i>	oi mi >	$\alpha > A, \omega > \Omega$

Figure 7: Neighborhoods, their icons, labels, mnemonics, correspondences, and constraints.

#### 4.1 Coarse reasoning based on neighborhoods

We have seen that the inferences that can be drawn from the composition table may be coarser than the initial conditions. We may have several reasons for stating the initial conditions for temporal inferences in coarser terms:

- 1) uncertainty may exist as to which initial condition in terms of the thirteen mutually exclusive relations holds;
- 2) the initial conditions may be stated in coarser terms corresponding to knowledge relating beginnings and/or endings of events (compare examples in section 2.1);
- 3) we want to use the conclusions from one inference step as initial conditions for further inference steps.

Coarser knowledge can always be expressed in terms of disjunctions of finer knowledge, inferences can be drawn on the basis of the finer knowledge, and conclusions can be derived by forming conjunctions of these inferences on a case by case basis. However, this approach is comparable to deriving general algebraic relationships from specific numerical instances. Rather than solving problems on the finest level of resolution we would like to solve them on the coarsest possible level.

In order to do this, we use as initial conditions neighborhoods of relations corresponding to disjunctions of fine relations which can be found as conclusions in the composition table. We select the neighborhoods in such a way that finer initial conditions can be expressed in terms of conjunctions of coarser initial conditions, if necessary. This step corresponds to aggregating neighboring Allen-relations. Initially, we do not aggregate  $<, m, m_i, >$ , since these relations do not have enough neighbors to form conjunctions for refinement. The ten relations and neighborhoods depicted in Figure 8 were selected. An interesting question is, how much knowledge about the corresponding temporal domain can be recovered from the coarse representation (compare ‘coarse coding’ in distributed representations, e.g.[13]).

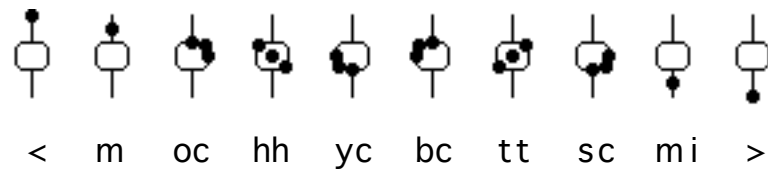


Figure 8: Ten relations and neighborhoods used as initial conditions for coarse reasoning.

We now form a composition table whose cell values consist of the disjunctions of all combinations of compositions of the constituent relations of the initial neighborhoods (Figure 9). Not surprisingly, we obtain similar patterns as in the case of fine reasoning. Due to the fact that we did not have any abrupt jumps between neighboring patterns, we only find connected neighborhoods again; due to the fact that in many cases neighboring patterns were sub-neighborhoods or super-neighborhoods of each other, we do not get many new patterns.

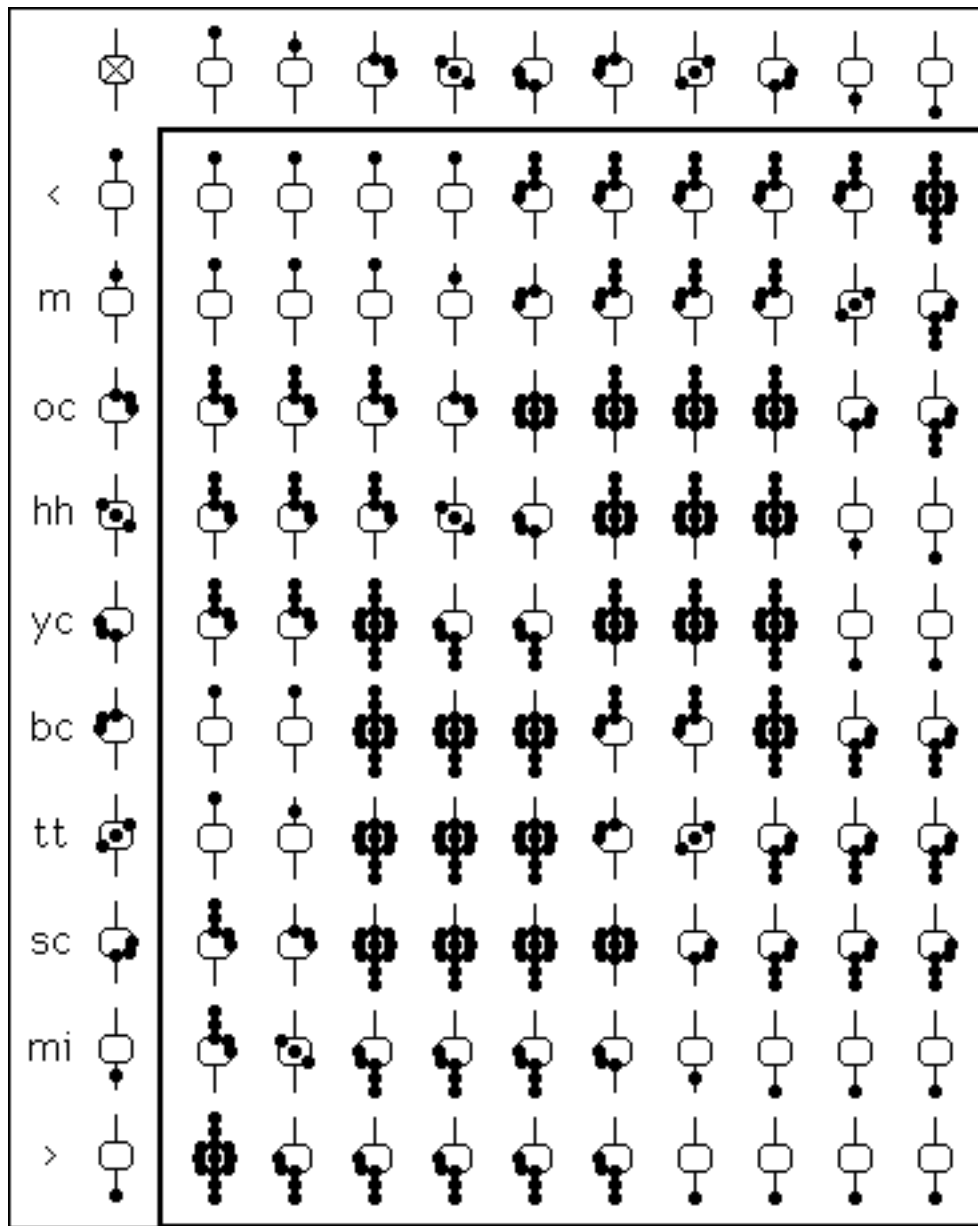


Figure 9: Neighborhood-based composition table condensed for coarse reasoning.

In fact, the patterns *ob* and *ys* disappeared and two new patterns corresponding to the disjunctions *ct ob* and *ct ys* appeared. These new neighborhoods will be called *born before death of (bd)* and *died after birth of (db)*, respectively (compare Figure 7).



The aggregated table shown in Figure 9 permits us to do two things: coarse reasoning and fine reasoning. For coarse reasoning, we simply look up the neighborhood of possible relations in the table. For example, if we know that X is an *older contemporary of* (OC) Y and Y is a *younger contemporary of* (YC) Z, we can infer that X is a *contemporary of* (CT) Z or if instead Y is *head to head with* (HH) Z then X is also an *older contemporary of* (OC) Z (see Figure 10). So, the conclusions do not necessarily become coarser when the initial conditions become coarser.

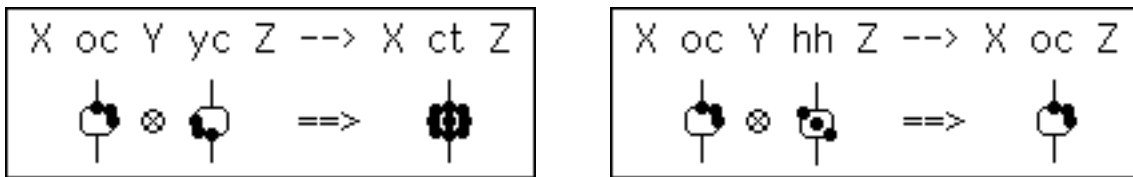


Figure 10: Two instances of the coarse composition relation (denoted by  $\otimes$ ).

#### 4.2 Fine reasoning based on neighborhoods

For fine reasoning, we form the conjunctions of the inferences we can draw by coarse reasoning. By algebraic considerations we obtain at least all fine relations which are obtained by fine reasoning. For example (see Figure 11), the fine relation  $X f Y$  corresponds to the two coarse relations  $X yc Y$  and  $X tt Y$ ; the fine relation  $Y o Z$  corresponds to the two coarse relations  $Y oc Z$  and  $Y bc Z$ . So, if  $X f Y$  and  $Y o Z$  hold, then the corresponding coarse relations also hold and so do the conclusions which we can draw from the interactions of the coarse relations. These interactions yield the neighborhoods  $?$ ,  $bd$ ,  $db$ ,  $bc$ ; the intersection of these neighborhoods is  $bc$ .

The result is identical to the result we get by fine reasoning. In fact, we obtain the correct optimal result in all cases. This is not due to the algebraic properties of the operations performed but due to the independence of constraints between the neighborhoods that are being

combined. For example,  $yc$  corresponds to the constraints  $\alpha > A$ ,  $\alpha < \Omega$  and  $tt$  corresponds to the independent constraint  $\omega = \Omega$  which combined yield the constraints  $\alpha > A$  and  $\omega = \Omega$ , which are the conditions for the relation  $f$  (compare Figure 1). The constraint  $\alpha < \Omega$  follows from  $\omega = \Omega$  and the domain-inherent constraint  $\alpha < \omega$ .

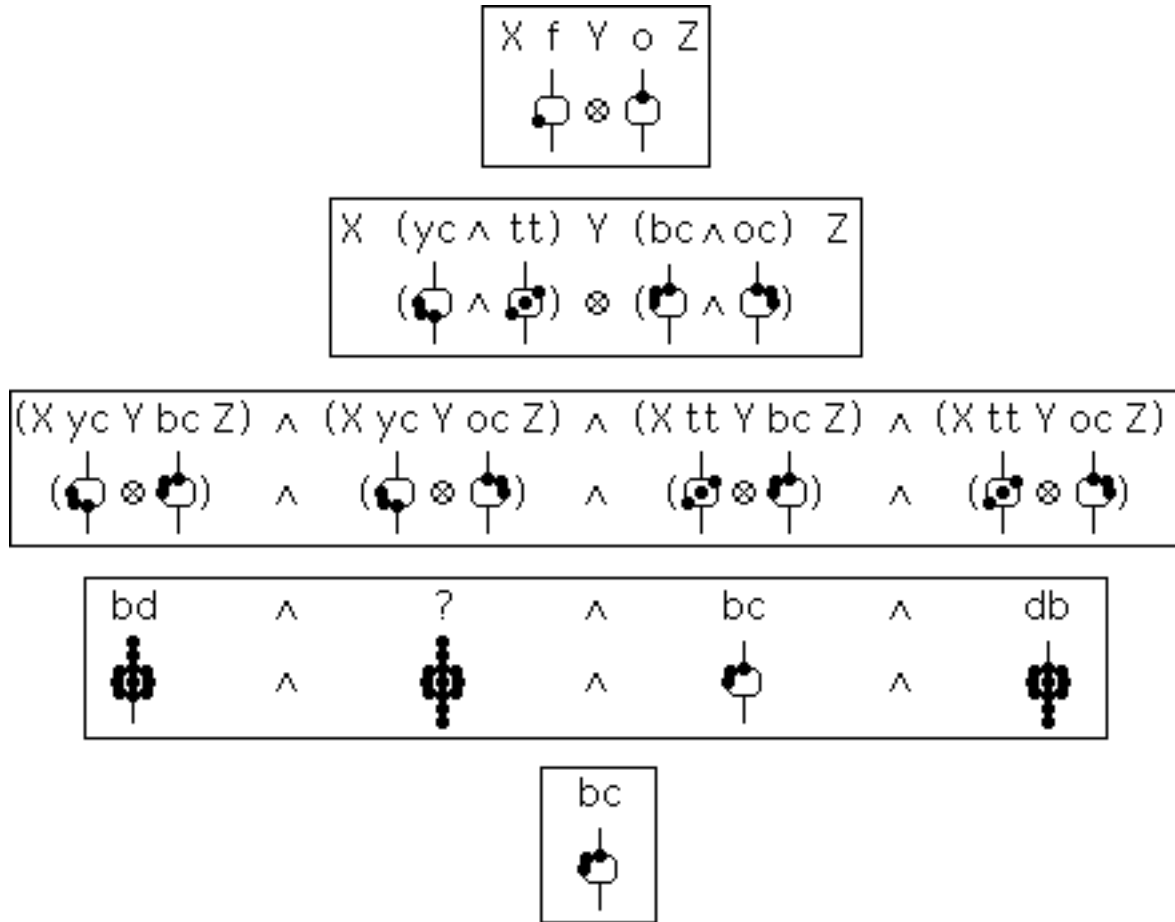


Figure 11: Elaborate fine reasoning by intersecting results from coarse reasoning.

In the same manner it is possible to combine fine knowledge (i.e., complete knowledge about the relation between two events) and coarse knowledge (here: knowledge about the relation between semi-intervals), in this inference scheme.

## 5 Inferential power and computational complexity

In this chapter, we first compare the inferential power of neighborhood-based temporal reasoning with that of Allen's approach and propose criteria for selecting appropriate composition tables. Then we discuss the subalgebra for neighborhood-based reasoning and apply complexity-theoretical results to this subalgebra.

### 5.1 Inferential power of neighborhood-based reasoning

If we consider the neighborhood-based composition table depicted in Figure 6, we easily can see that it has all the inferencing capabilities of Allen's original table: the represented knowledge in both tables is identical; only the arrangement differs. The new arrangement together with the monotonicity properties described in chapter 4 yields additional reasoning capabilities: the table can be used for interpolation between known conclusions and for predicting conclusions in the case of uncertain initial conditions.

What happens with the inferential power when we condense the composition table for coarse reasoning (Figure 9)? First of all, all inferences that can be drawn with Allen's table still can be drawn and yield identical results. However, inferences based on the fine relations used by Allen can become computationally more expensive: in 81 of 169 possible inferences, a simple table look-up is replaced by a conjunction of four table look-ups.

Reasoning with the condensed table, however, is cheaper when coarser knowledge is involved. The computational pay-off is best when the processed knowledge grains agree in size and shape with the neighborhoods represented in the composition table. This minimizes the number of conclusions to be computed and combined by disjunctions and/or conjunctions. For example, for the central part of the condensed composition table (granularity 3), an inference from two neighborhood triplets involves a single table look-up instead of forming the disjunction of nine individual look-ups, as with Allen's table.

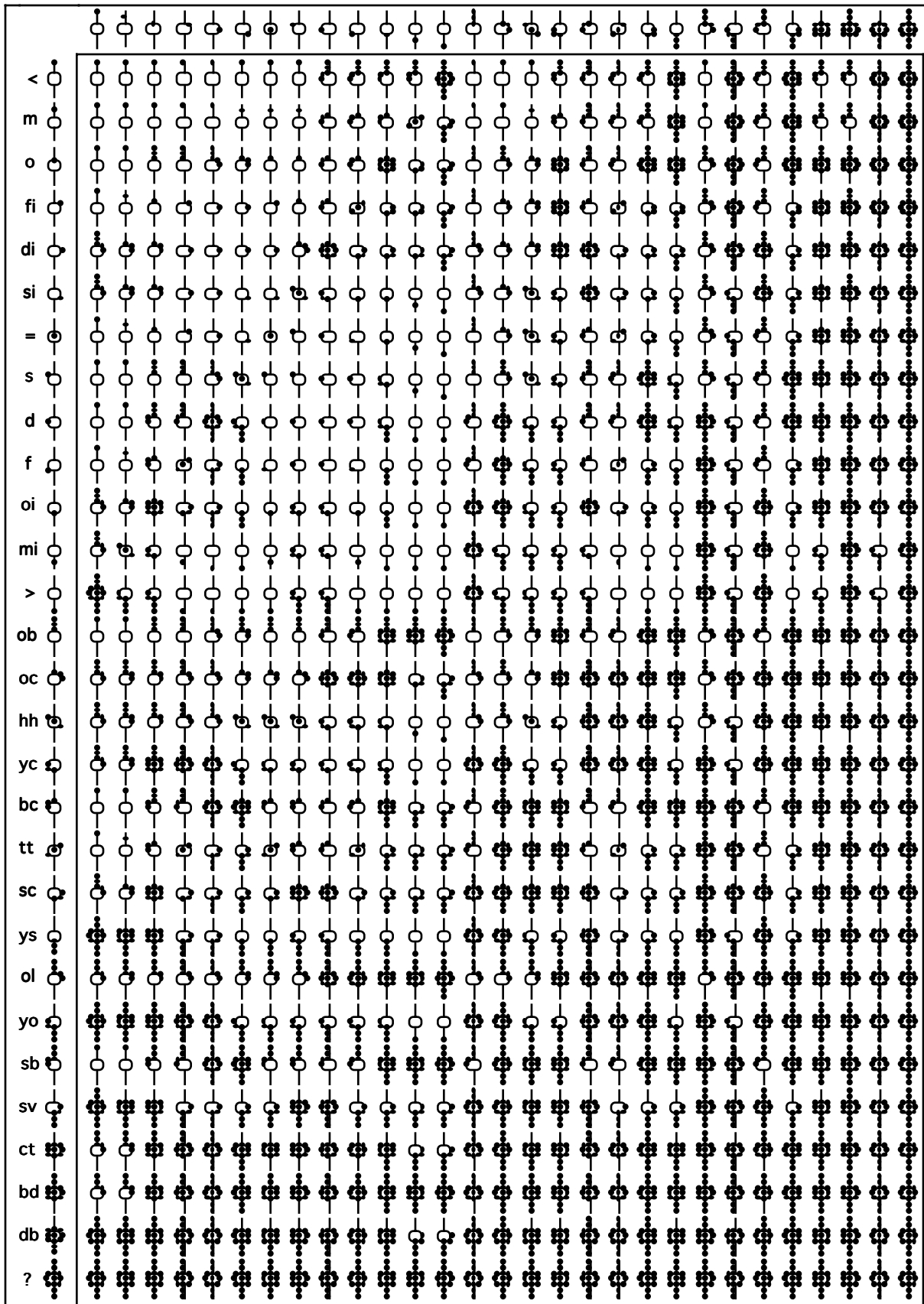
Thus, the condensed table shifts computational effort and yields additional inferencing capabilities. For processing fine knowledge, Allen's composition table is advantageous, for processing coarser knowledge, the condensed table works more efficiently. In general, knowledge processing becomes more efficient when it can be shifted to a coarser level of processing: one coarse inference can do the work of nine fine inferences, under favorable conditions.

There are two ways of combining temporal inferences: 1) propagating inferred knowledge along an inference chain by the composition operation; here knowledge tends to become coarser – by a factor of 2.4 per operation, in the average; 2) combining knowledge from multiple evidence sources by forming the logical conjunction; here knowledge tends to become finer by the same order of magnitude – precise values depend on the specific data involved. Depending on the granularity of the inference table used, the sequence of propagating knowledge from a single source or combining knowledge from multiple sources can be adapted in order to optimize the knowledge granularity for the given table.

Depending on the aspects to be optimized in a given application, we can conceive of a variety of different inference tables from a compact table requiring disjunctions and/or conjunctions of inferences to an elaborate table representing the closed set of relations generated by the composition of the 13 fine relations. This table consists of  $29 \times 29$  entries (the 13 fine relations plus the relations shown in Figure 7, except *pr* and *sd*, which do not occur in the inference tables discussed so far). The resulting table is closed under neighborhood-based reasoning and does not require disjoining or conjoining neighborhoods for knowledge propagation. The table is shown in Figure 12.

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Next page: Figure 12: 29 convex *A-neighbor* relations forming a closed set under composition.



As the discussion has shown, efficiency can be improved by ‘tuning’ the inference tables, but this is not a big issue – at least not in the simple domain of temporal reasoning; the factor of improvement is unlikely to exceed 3, for typical applications. A more substantial result concerns the complexity of computing the closure for neighborhood-based reasoning. This topic will be addressed in the following section.

## 5.2 Neighborhoods and convex relations

Allen’s polynomial time algorithm for temporal reasoning never infers invalid consequences from a set of assertions, but it does not guarantee that *all* the inferences that follow from the assertions are generated; thus the algorithm is incomplete. Vilain and Kautz have shown that computing the closure in the full interval algebra is an NP-complete problem (which only can be solved in exponential time) [20, 21].

Vilain, Kautz, and van Beek [21] and Nökel [19] discuss a subset of Allen’s full interval algebra which has a tractable closure algorithm, i.e., closure can be computed in polynomial time. This subset is defined by a property of semi-interval relations which Vilain et al. call ‘continuous endpoint uncertainty’. Continuous endpoint uncertainty is a convexity property and means that for any two interval end points belonging to a common semi-interval relation, intermediate end points belong to the relation as well.

Vilain et al. define continuous endpoint uncertainty for the relation between time points. They apply this definition to the relation between intervals by considering individual relations between the beginnings and endings of two intervals. By this method, the continuous uncertainty property generates the set of ‘convex interval relations’ [19] on the structure defined by the *A-neighbor* relation in section 3.2 (Figure 4). 808 of the 8191 possible interval relations form *A-neighborhoods* (i.e. neighborhoods under the *A-neighbor* structure). 82 of these neighborhoods are convex relations in this structure and form the tractable algebra discussed above. The closed set of 29 neighborhoods generated by the 13 fine interval relations under

composition, in turn, forms a subalgebra of the algebra of convex relations; thus it is tractable as well.

When continuous endpoint uncertainty is applied simultaneously to pairs of relations between beginnings and endings of two intervals, different neighborhood structures evolve: we obtain the *B-* and *C-neighbor* relations depicted in Figure 4. Additional relations become neighbors ( $o =$ ,  $oi =$ ,  $d =$ , and  $di =$ ); they were only indirect neighbors under the *A-neighbor* relation (via a chain of two direct neighbor links). The pairs of relations  $s =$ ,  $f =$ ,  $si =$ ,  $fi =$  are not neighbors in the *B-* and *C-neighbor* relations.

There are 769 *B-neighborhoods* and 529 *C-neighborhoods*; 1255 neighborhoods are obtained by combining the three types of neighbor relations. Some of the 29 convex relations forming a closed set under composition are not *B-* or *C-neighbors* ( $hh$  and  $tt$ ). For the disjunction of the *A-*, *B-*, and *C-neighbor* relations, the strict convexity property disappears for the closed set of 29 neighborhoods: for example, the conceptual neighborhood  $o s d$  is not convex without the relation  $=$  under the disjunction of *A-*, *B-* and *C-neighbor* relations.

Nevertheless, the *B-* and *C-neighbor* relations and their combination with the *A-neighbor* relation are useful for neighborhood-based reasoning: recall that the monotonicity properties described in chapter 4 hold for all three neighborhood relations and thus may be used for the interpretation of the conclusions drawn on the basis of the neighborhood subalgebra.

In summary, real-world constraints on temporal events and their interrelationships have allowed us to condense temporal knowledge by removing redundancies; as a side-effect, temporal reasoning becomes more efficient. The structure obtained in this process turns out to be an interesting subset of the full interval algebra: 1) it is a natural subset generated by the composition operation as the closure of the basic temporal relations; 2) it corresponds to an important class of physical situations; and 3) it is computationally tractable. In the following

chapter, we will discuss additional regularities of temporal relationships which may amplify our understanding of temporal structures.

## 6 Compacting the knowledge base

The smoothness of the transitions between neighborhoods allowed us to aggregate temporal relations for neighborhood-based reasoning. By aggregating relations, the composition table shrank from  $13 \times 13 = 169$  to  $10 \times 10 = 100$  entries (Figure 9). As we will show in the present chapter, we are able to further simplify the knowledge base underlying the reasoning scheme.

In the example given in section 4.2 we showed how fine knowledge can be obtained by combining intersecting pieces of coarse knowledge. The final conclusion we obtained in the inference process was already present in full detail in one of the four inferences we combined, namely in the inference drawn from  $X \text{ tt } Y$  and  $Y \text{ bc } Z$ . Do we always have to form the conjunction of all possible inferences from the intersecting initial neighborhoods or can we systematically simplify the procedure?

Inspection of the inferences based on the condensed composition table (Figure 9) shows that in no case more than two sub-inferences contribute towards the solution of the full inference. In fact, only 54 of the 100 entries of the table yield useful constraints for the reasoning procedure. These entries are depicted in Figure 13.



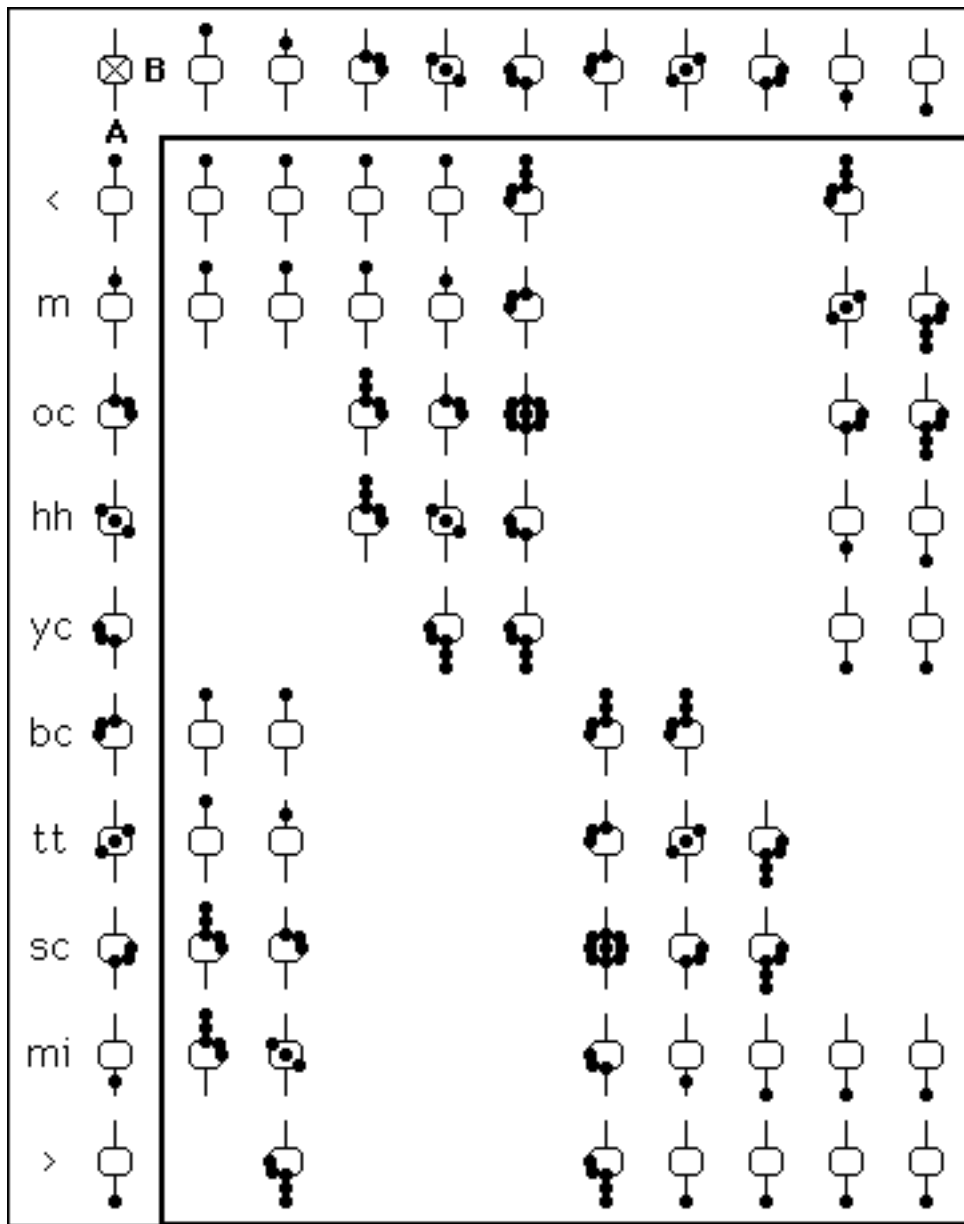


Figure 13: Condensed composition table without non-contributing entries.

### 6.1 Symmetry and redundancy

There is quite a bit of symmetry in the neighborhood-based composition table which can be exploited for matrix simplification. Most obvious is the symmetry between the top and bottom halves of the table: if for both initial conditions A and B (compare Figure 13) the icons are flipped vertically, the table entries are flipped vertically. This corresponds to the symmetry between '<' and '>' when comparing semi-intervals.

After compaction of the table due to this symmetry, the columns corresponding to the neighborhoods *bc*, *tt*, *sc* are not needed and can be eliminated. In addition, the first two columns can be merged. This corresponds to forming a neighborhood of the relations *<* and *m*. Figure 14 shows the table compacted to 25 entries. On the right hand and bottom sides of the table the initial conditions for the entries to be vertically flipped are shown.

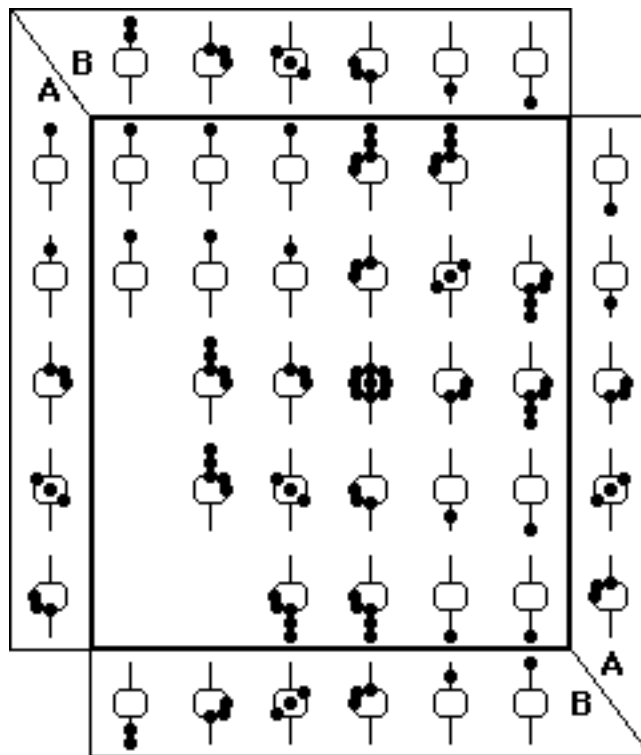


Figure 14: Transitivity table compacted to 25 entries.

Further symmetries and other regularities allow the elimination of all but seven table entries: The entries in the upper right hand corner of the table shown in Figure 14 can be mapped into the lower left half of the table by exchanging the x- and y-axes of the initial conditions and by flipping the corresponding entries horizontally. This transformation yields layers II - ii of initial conditions (Figure 15).

The entries in the lower right hand corner of the table in Figure 14 can be mapped into the upper left half of the table by inverting the x- and y-axes of the initial conditions and flipping the table entries both vertically and horizontally (or equivalently, by rotating them by 180 degrees). This transformation yields layers III - iii and IV - iv.

Furthermore, we find four identical entries in the upper left hand corner of the table. They can be mapped into a single entry by adding layers V - v, VI - vi, and VII - vii, each containing one singleton of initial conditions.

Finally, the table can be simplified in order to minimize transformations on the table entries; this is done by rotating the remaining entries by 180 degrees.

Figure 15 shows the compressed table consisting of seven entries which are accessible by  $2 \times 7$  layers of initial conditions. Two of the entry patterns are identical (yo). The initial conditions in each layer are mutually exclusive. Neighboring initial conditions always correspond to neighboring neighborhoods, in each layer. Only pairs of conditions belonging to the same layer (I-VII or i-vii) have to be used for accessing the entries. Entries derived from layers marked with a vertical double arrow have to be flipped vertically, entries derived from layers marked with a horizontal double arrow have to be flipped horizontally, entries derived from layers marked with both arrows have to be flipped in both directions.

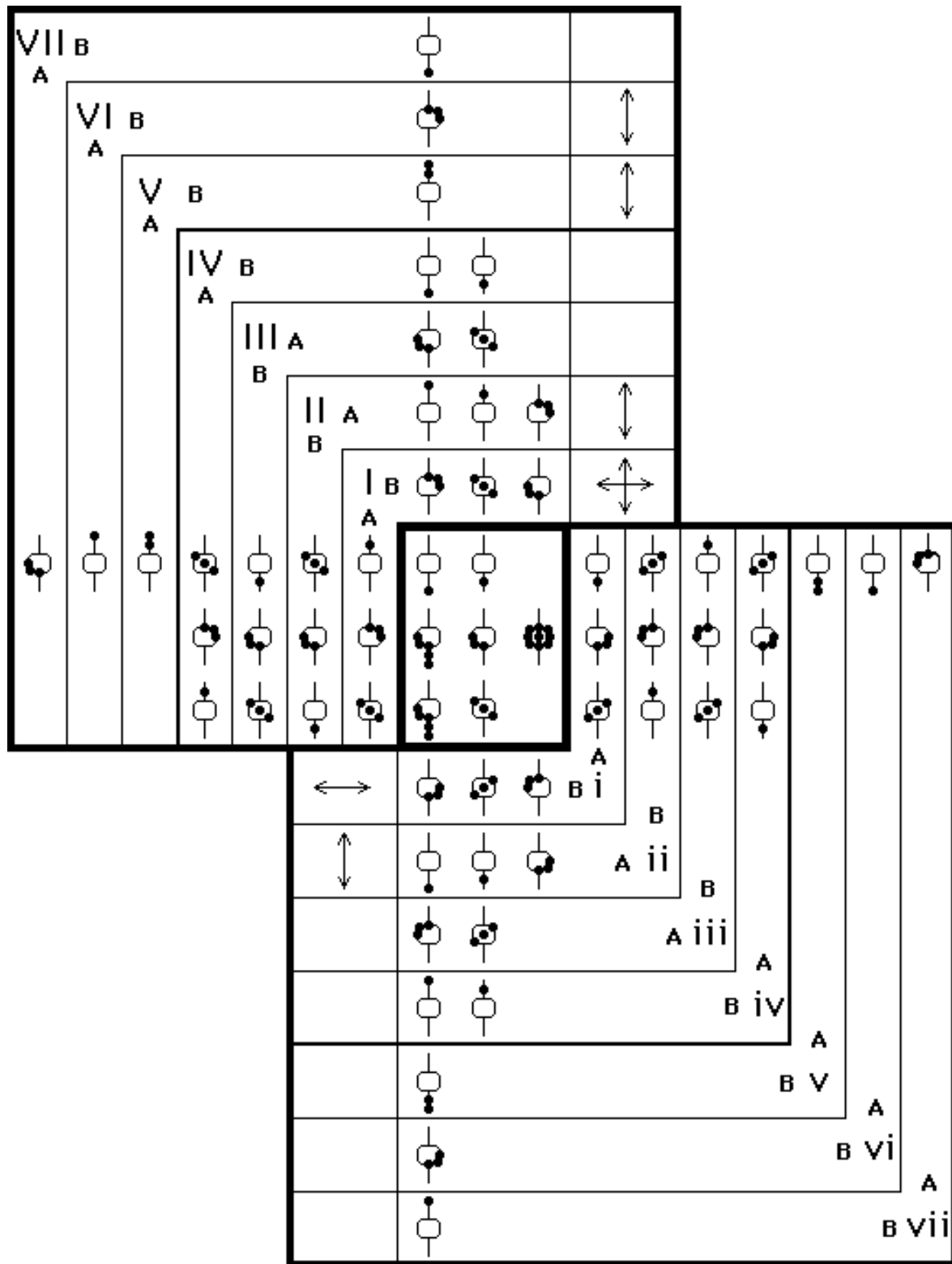


Figure 15: Transitivity table compressed to 7 entries.

### 6.2 Reasoning based on the compressed composition table

The compressed composition table represents very general regularities corresponding to the symmetries involved in the relations between neighborhoods. Only the first 2\*4 layers of initial conditions (I - IV and i - iv) actually define the structure of the table; the other 2\*3 layers (V - VII and v - vii) all refer to the same single entry in the table.

In order to use this table for temporal reasoning, neighborhoods of event relations and/or individual event relations are matched with the corresponding initial conditions for the table as in the previous composition tables. Then the conjunction of the corresponding entries is formed. The entries corresponding to the initial conditions marked with arrows must be flipped as suggested by the arrow before the conjunction is formed.

Flipping the entry patterns corresponds to a very simple re-labeling of relations. Specifically, horizontal flipping corresponds to exchanging the labels  $f_i$  and  $s$ ,  $d_i$  and  $d$ ,  $s_i$  and  $f$ ; vertical flipping corresponds to exchanging the labels  $<$  and  $>$ ,  $m$  and  $m_i$ ,  $o$  and  $o_i$ ,  $f_i$

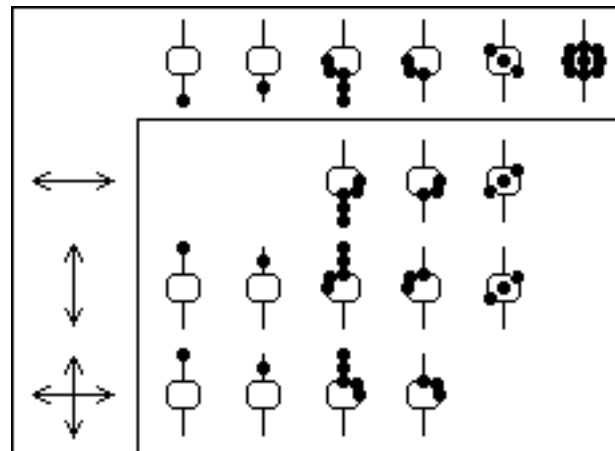


Figure 16: The effect of horizontal and vertical flipping of the 6 distinct entries of the compressed composition table. Blank entries indicate that the transformation has no effect and is therefore not required.

and  $s_i$ ,  $s$  and  $f$ ; flipping both dimensions corresponds to exchanging the labels  $<$  and  $>$ ,  $m$  and  $m_i$ ,  $o$  and  $o_i$ ,  $f_i$  and  $f$ ,  $d_i$  and  $d$ ,  $s_i$  and  $s$  (compare Figure 16).

### 6.3 Examples for reasoning with the compressed composition table

1) Fine reasoning. Suppose,  $X$  is *started by*  $Y$  and  $Y$  *finishes*  $Z$ . What is the relationship between  $X$  and  $Z$ ? We check the layers of initial conditions for pairs  $A$ ,  $B$  corresponding to the pair  $s_i$ ,  $f$ . We obtain four matches: a) layer I: bottom right entry; b) layer i: center entry; c) layer ii: top right entry; d) layer III: center entry. a) and c) correspond to non-contributing entries; thus, we only have to consider b) and d). Both sets of initial conditions point to the center entry of the table corresponding to relation  $yc$ . The table indicates that entries associated with layer i have to be flipped horizontally; therefore we form the conjunction of  $yc$  and its horizontally flipped image  $sc$ . We obtain  $o_i$ . Thus,  $X$  is *overlapped by*  $Z$  is the final conclusion.

2) Coarse reasoning. Suppose,  $X$  is a *younger contemporary of*  $Y$  and  $Y$  is *head to head with*  $Z$ . Layer III contains the matching pair of initial conditions which point to the relation *younger*. Layer III does not indicate that flipping is required; thus  $X$  is *younger* than  $Z$  is the final conclusion.

3) Combining fine and coarse knowledge. Suppose,  $X$  *meets*  $Y$  and  $Y$  is a *younger contemporary of*  $Z$ . How are  $X$  and  $Z$  temporally related? We check the layers of initial conditions for pairs  $A$ ,  $B$  of initial conditions corresponding to the pair  $m$ ,  $yc$ . We obtain two matches: top right entry for layer I and center entry for layer II. The top right entry is a non-contributing entry, so we only have to consider the center entry of the table which corresponds to the neighborhood  $yc$ . The table indicates that the entries obtained through layer II have to be vertically flipped; this yields the relation  $bc$ . Thus,  $X$  is a *survived by contemporary of*  $Z$  (compare Figure 17).

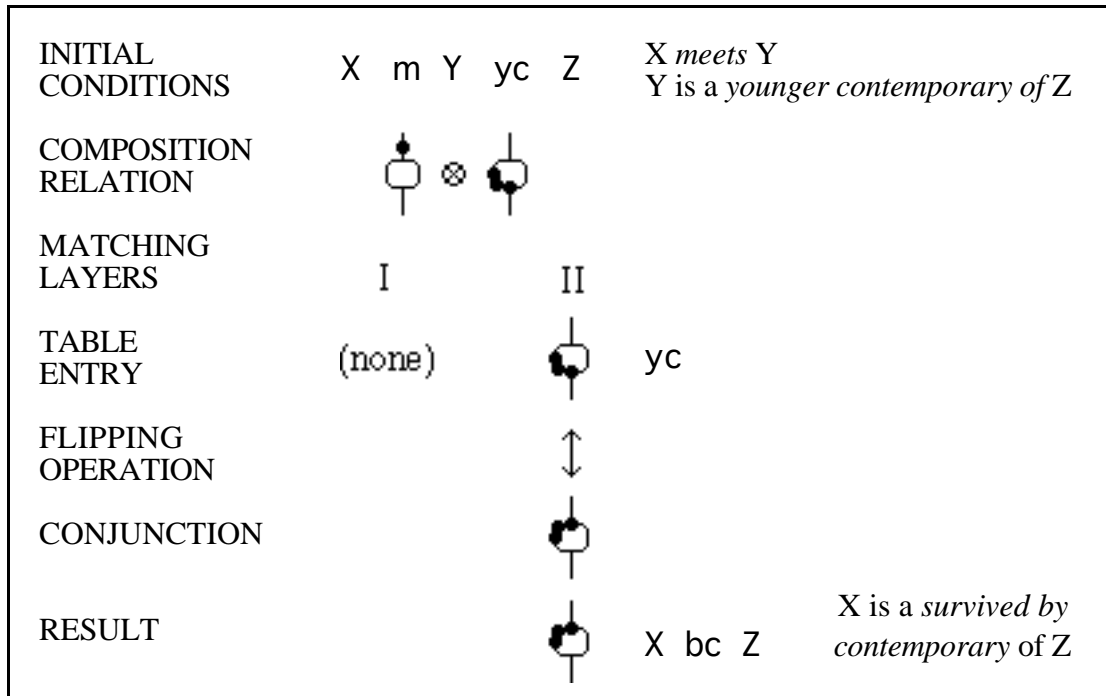


Figure 17: The reasoning steps involved in reasoning with the compressed composition table. The example shows how fine and coarse knowledge can be combined.

## 7 Conclusions

We have modified Allen’s approach to interval-based representation of temporal relations in such a way that it can be used rather naturally for reasoning with incomplete knowledge, specifically with coarse knowledge about temporal relationships. Our approach adds flexibility and appears to be cognitively more adequate. It is based on a neighborhood-oriented view of events: events are not treated as isolated entities; rather, they are viewed as conceptual items which are embedded in a network of related events. In this view, the notion of ‘conceptual neighborhood’ becomes essential.

Conceptual neighborhood plays an important role in cognition. Many cognitive functions rely on the assumption that the world they are dealing with is continuous or quasi-continuous, i.e., changes happen in steps rather than in jumps. For the specific domain of temporal relations we have shown that this assumption is justified.

The concept of neighborhood is a prerequisite for our concept of coarse knowledge. Coarse knowledge allows for short-cuts in reasoning in the following way. Allen's original reasoning strategy conceptually contains four levels of knowledge: 1) problem level in terms of coarse knowledge; 2) initial conditions expressed in terms of fine knowledge; 3) constraints on the composition relation corresponding to coarse knowledge; 4) conclusion expressed in terms of fine knowledge. Level 1) is present only if the problem is initially given in coarse terms; level 4) is present only if the result is stated in fine terms. In our approach, we merge levels 1) to 3) by reasoning directly on the coarse level.

I would like to suggest that this short-cut is just one instance of neighborhood-based problem reduction and that the general idea can be applied in various domains of cognition. For example, in natural language representation, concepts are frequently represented in fine terms; as a consequence, semantical ambiguities demand a multiplication of processing effort. If the concepts were represented on a higher conceptual level, some of the ambiguities would never arise and consequently would not have to be resolved. Another domain is theorem proving<sup>2</sup> where it is desirable to identify coarser concepts whose conceptual instants share important properties.

The neighborhood-based inference strategy described in this article has been implemented and compared with with Allen's strategy [17], but a large scale performance analysis under various conditions has not yet been done. We view the neighborhood structures described as basic generic structures for the construction of very small sequential reasoners and of regularly

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<sup>2</sup> this suggestion is due to Steffen Hölldobler.



structured parallel reasoners. For a sequential reasoner, the compressed composition table would be sequentially accessed by the layers matching the initial conditions before the entries are flipped and conjoined. For a parallel reasoner, several copies of the table could be accessed simultaneously; the entry-flipping could be wired-in directly. Also, the simple and regular structure of the neighborhood reasoning structure (Figure 9) appears to make implementation by means of an associative memory appropriate.

The presented approach can be extended in various directions. The neighborhood concept can be used for reasoning under uncertainty. Uncertainty or incomplete knowledge correspond to a neighborhood of possibilities compared to a single possibility within this neighborhood in the case of certainty or complete knowledge. This view is in contrast to a view in which uncertainty or incomplete knowledge correspond to disjunctions of unrelated possibilities. If coarse knowledge becomes coarser due to fuzziness, the same reasoning principles can be applied to coarse knowledge which we have applied to fine knowledge; although full recovery of fine knowledge will no longer be guaranteed.

An obvious extension of the approach is for reasoning with 1-dimensional space which shares many properties with time. Extensions for reasoning about 2- or 3-dimensional space are more challenging (compare [4]), but a coarse reasoning approach appears to be better tractable than a fine reasoning approach. We expect that the large amount of regularity and the conceptual simplicity of the system will prove helpful for developing representation schemes for more-dimensional spaces.

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