ELSEVIER

Information Sciences 143 (2002) 47–71

INFORMATION SCIENCES
AN INTERNATIONAL JOURNAL

www.elsevier.com/locate/ins

Temporal granulation and its application to signal analysis

Witold Pedrycz [a,*], Adam Gacek [b]

[a] *Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2G7*

[b] *Institute of Medical Technology and Equipment (ITAM), 118 Roosevelt st., Zabrze 41-800, Poland*

Received 16 November 1999; received in revised form 18 February 2001; accepted 29 May 2001

Abstract

In this study, we elaborate on the role of information granulation and the ensuing information granules in description of time series and signal analysis, in general. Information granules are entities of elements (quite commonly, numeric data) that are combined together (aggregated) owing to their vicinity, similarity and alike. Proceeding with a given window of granulation (that is an initial collection of numeric data), we propose an algorithm that produces a complete information granule – fuzzy set. The principle supported by the method leads to the formation of fuzzy sets that are legitimate in terms of experimental data being at the same time maximized with regard to their specificity (compactness). It has been shown that information granules can be are regarded as generic conceptual entities contributing to the description of numeric time series. In this capacity, they are used as building blocks aimed at achieving high level, compact, and comprehensible models of signals. More importantly, the phase of information granulation could be viewed as a prerequisite to more synthetic and abstract processing such as the one witnessed in syntactic pattern recognition

Keywords: Information granularity; Temporal and spatial granulation; Fuzzy sets; Temporal granulation; Signal processing; ECG signal analysis; Syntactic pattern classifiers

[*] Corresponding author. Tel.: +1-780-492-3333; fax: +1-780-492-1811.
E-mail address: pedrycz@ee.ualberta.ca (W. Pedrycz).

0020-0255/02/$ - see front matter
PII: S0020-0255(02)00179-2

1. Introduction

Information granules [2,18–20,13] are viewed as linked collections of objects (data points, in particular) drawn together by the criteria of indistinguishability, similarity or functionality. Information granules and the ensuing process of information granulation is a vehicle of abstraction leading to the emergence of high-level concepts.

Granulation of information is an inherent and omnipresent activity of human beings carried out with intent of better understanding of the problem and coming up with an efficient problem-solving strategy. In particular, granulation of information is aimed at splitting the problem into several manageable chunks. In this way, we partition the problem into a series of well-defined subproblems (modules) of a far lower computational complexity than the original one we have started with.

Granulation occurs everywhere; the examples of granulating information are abundant:

- We granulate information over time by forming information granules over predefined time intervals. This gives rise to the notion of *temporal* granulation. For instance, one computes a moving average with its confidence intervals.
- In any computer model we granulate memory resources by subscribing to the notion of pages of memory as its basic operational chunks (then we may consider various swapping techniques to facilitate an efficient access to individual data items).
- We granulate information available in the form of digital images – the individual pixels are arranged into larger entities and processed as such. We usually refer to these activities as *spatial* granulation. This leads us to various issues of scene description and analysis.
- In describing any problem, we tend to shy away from numbers but rather start using aggregates and building rules (*if–then statements*) that dwell on them.
- We live in an inherently analog world. Computers, by tradition and technology, perform processing in a digital world. Digitization of this nature (that dwells on set theory-interval analysis) is an example of information granulation.
- All mechanisms of data compression are examples of information granulation that is carried in a certain sense.

Overall, there is a profound diversity of the situations that call for information granulation. There is also panoply of possible formal vehicles to be used to capture the notion of granularity and provide with a suitable algorithmic framework in which all granular computing can be efficiently completed. In the ensuing section, we elaborate on those commonly encountered in the literature. Examples of such formal environments include set theory, rough sets [11], random sets, shadowed sets [12] or fuzzy sets [18].

The environment of fuzzy set technology is of particular interest. Fuzzy sets offer two interesting and useful features supporting processes of information granulation and the form of information granules resulting therein. First, fuzzy sets support modeling of concepts that exhibit continuous boundaries. The overlap between fuzzy sets (that is an inherent phenomenon occurring in the theory of fuzzy sets) helps avoid a brittleness effect manifesting when moving from one concept to another. This becomes particularly crucial in the case of using data that could be affected by some noise. The noisy counterpart may then have a very profound impact on the performance of any rule-based architecture. Second, fuzzy sets exhibit a well-defined semantics and emerge as fully meaningful conceptual entities – building modules identified in problem solving [3].

The material is organized into eight sections. We discuss in Section 1 the essence of information granulation and its realization in the setting of fuzzy sets (Section 2). Then, in Section 3, we elaborate on some interesting properties of general classes of membership functions by addressing an issue of their sensitivity and a distribution of this property along the range of membership values. The two subsequent sections deal with the development of the data-justifiable fuzzy sets by presenting the underlying concept and discussing a detailed algorithm. Section 6 is devoted to two selected applications of granular data to signal processing such as granular predictive models and an idea of condensation of numeric signals and their graph representation. Section 7 includes conclusions. In this study, we consider synthetic as well as real-world data sets. The first ones help illustrate the underlying idea. The second group of data sets comes the MIT-BIH database of ECG signals.

2. Information granulation with the use of fuzzy sets

There are three main ways in which information granules – fuzzy sets or fuzzy relations can be constructed:

- User-oriented. It is a user or designer of the system who completely identifies the form of the information granules. For instance, they could be a priori defined as a series of triangular fuzzy numbers. Moreover, the number of these terms as well as their parameters are fully specified in advance.
- Algorithmic approach to information granulation. In this case, information granules come as a result of optimization of a certain performance index (objective function). Clustering algorithms are representative examples of such algorithms of unsupervised learning leading to the formation of the information granules. Quite commonly, the granules are fuzzy sets (or fuzzy

relations) when using FCM and alike or sets (or relations) when dealing with the methods such as ISODATA [1,3,5].

- A combination of these two. The methods that fall under this category come as a hybrid of user-based and algorithmic driven methods. For instance, some parameters of the information granulation process can be set up by the user while the detailed parameters of the information granules can be determined (or refined) through some optimization mechanism that is used during the second phase. The amount of influence coming from the user and data varies from case to case.

One should become aware of the advantages and potential drawbacks of the two first methods (the third one is a compromise between the user and data-driven methods and as such may reduce the disadvantages associated with its components). The user-based approach, even quite appealing and commonly used, may not reflect the specificity of the problem (and, more importantly, the data to be granulated). There could be a serious danger of forming fuzzy sets not conveying any experimental evidence. In other words, we may end up with a fuzzy set whose existence could be barely legitimized in light of the currently investigated data. The issue of the experimental legitimization of fuzzy sets along with some algorithmic investigations has been studied in detail in [14]. On the other hand, the algorithmic-based approach could not be able to reflect the semantics of the problem. Essentially, the membership functions are built as constructs minimizing a given performance index. This index itself may not capture the semantics of the information granules derived in this fashion. Moreover, the data-driven information granulation may be computationally intensive especially when dealing with large sets of multidimensional data (that are common to many tasks of data mining). This may eventually hamper the usage of clustering as a highly viable and strongly recommended option in data mining.

Bearing in mind the computational facet of data mining, we consider a process of granulation that takes place for each variable (attribute) separately. There are several advantages to follow this path. First, the already mentioned computational aspect being essential in data mining pursuits is taken care of. Second, there is no need for any prior normalization of the data that could eventually result in an extra distortion of relationships within the database; this phenomenon has been well known in statistical pattern recognition [6,7]. The drawback of not capturing the relationships between the variables can be considered minor in comparison to the advantages coming with this approach. In building a series of information granules we follow the hybrid approach, namely we rely on data but provide the number of the linguistic terms in advance along with their general form (type of membership function).

Before proceeding with the complete algorithm, it is instructive to elaborate on different classes of membership functions and analyze their role in information granulation.

3. Classes of membership functions and their characterization

There is an abundance of classes of membership functions encountered in the theory and applications of fuzzy sets. Several useful guidelines as to a suitable selection are worth underlining:

- Information granules need to be flexible enough to "accommodate" (reflect) the numeric data. In other words, they should capture the data quite easily so that the granule become *legitimate* (viz. justifiable in the setting of experimental data). This implies membership functions equipped with some parameters (so that these could be adjusted when required). The experimental justification of the linguistic terms can be quantified with the aid of probabilities, say the probability of the fuzzy event [12,20] Given a fuzzy set A, its probability computed in light of experimental data $\mathbf{X} = \{x_1, x_2, .., x_N\}$ originates as a sum of the membership values.
-
$$\mathrm{Prob}(A) = \sum_{k=1}^{N} A(x_k)/N,$$
  we say A is *experimentally justifiable* if the above sum achieves or exceeds a certain threshold value γ.
- Information granules need to be "stable" meaning that they have to retain their identity in spite of some small fluctuations occurring within the experimental data. This also raises a question of sensitivity of the membership functions and an issue of its distribution vis-à-vis specific values of the membership grades. We claim that the sensitivity of the membership values should be more evident for higher membership grades and decay for lower membership grades. This is intuitively appealing: we are not concerned that much about the lower membership values while the values close to 1 are of greater importance as those are the values that imply the semantics of the information granule. The quantification of this property can be done by the absolute value of the derivative of the membership function A regarded as a function of the membership value (u), namely
$$s(A)(u) = \left| \frac{\mathrm{d}A(x)}{\mathrm{d}x} \right| = \varphi(u).$$

In what follows, we analyze three classes of membership functions such as triangular, parabolic, and Gaussian fuzzy sets by studying the two criteria established above.

The triangular fuzzy sets are composed of two segments of linear membership functions, see Fig. 1(a). The membership function reads as

$$\mathbf{T}(x; a, m, b) = \begin{cases} \frac{x-a}{m-a} & x \in [a, m], \\ 1 - \frac{x-m}{b-m} & x \in [m, b] \end{cases}$$

(for the rest of the arguments, the membership values are equal to zero).

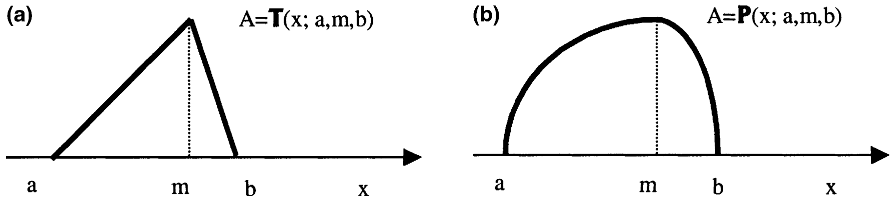


Fig. 1. Two general classes of triangular (a) and parabolic membership functions (b).

It is defined by three parameters that is a modal value (m) and two bounds (a and b). The left- and right-hand of the fuzzy set are determined separately. Then the parametric flexibility is available in the design of the information granule. The sensitivity of A is constant and equal to the increasing or decreasing slope of the membership function. The sensitivity does not depend on the membership value and does not contribute to the stability of the fuzzy set.

The parabolic membership functions are defined by three parameters (a, m, and b), Fig. 1(b)

$$\mathbf{P}(x; a, m, b) = \begin{cases} 1 - \left(\frac{x-m}{m-a}\right)^2, & x \in [a, m], \\ 1 - \left(\frac{x-m}{b-m}\right)^2, & x \in [m, b], \\ 0 \quad \text{otherwise.} \end{cases}$$

These parameters can make the fuzzy set asymmetrical and help adjust the two parts of the information granule separately. The sensitivity exhibits an interesting pattern; it achieves the highest values around the membership value equal to 1 and tempers down to zero when the membership values approach zero. In this sense, the range of high-membership values of the granule becomes emphasized, Fig. 2.

The Gaussian membership function is governed by the expression

$$A(x) = \exp(-(x-m)^2/\sigma^2)$$

and includes two parameters (m and σ). The first one (m) determines the position of the fuzzy set. The second parameter (σ) controls the spread of the information granule. Gaussian fuzzy sets are symmetrical. This may form a certain problem when the data exhibit a significant asymmetry that cannot be easily copied with. The sensitivity pattern exhibits its maximum around the membership value equal to 0.5, see Fig. 3.

4. The development of data-justifiable information granules: a general strategy

Our objective is to construct fuzzy sets that are legitimized by data. The problem is posed in the following way:

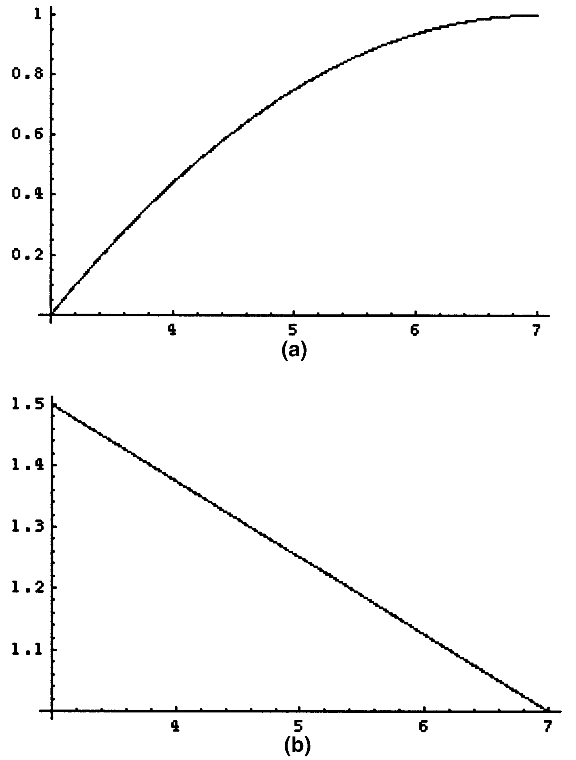


Fig. 2. Sensitivity pattern of the parabolic membership function with $m = 7$; $a = 3$; shown is an increasing portion of the membership function (a) and its derivative (b).

- Given is a collection of numeric one-dimensional data $\mathbf{X} = \{x_1, x_2, \ldots, x_N\}$ where x_k is a real number. This collection will be also referred to as Construct a fuzzy set A belonging to a certain family of fuzzy sets A (say, triangular, parabolic, etc.) so that it "legitimized" in the sense of its experimental evidence while being specific enough so that it support is kept small. The above formulation of the problem has a strong intuitive underpinning. On one hand, we want the fuzzy set to embrace enough experimental evidence. On the other hand, the information granule should become specific enough. These two requirements, that are conflicting to some degree, can be articulated in the following manner.
- Maximize the sum of membership values.
- $$\sum_{k=1}^{N} A(x_k)$$

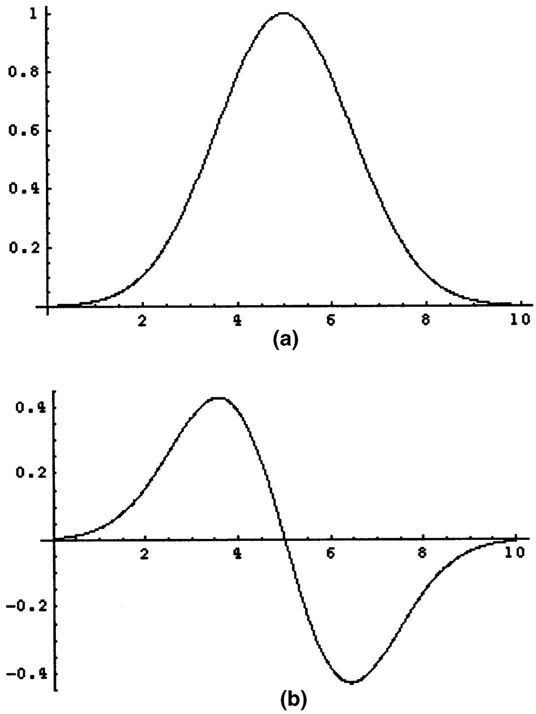


Fig. 3. Sensitivity pattern of the Gaussian membership function with $m = 5$; $s = 2$; membership function (a) and its derivative (b).

(note that the above is just proportional to the probability of the fuzzy event A manifested through the discrete data $\mathbf{X}$).

- Minimize the support of the fuzzy set that leads to higher specificity

$$\text{measure}(\text{supp}(A)) = (b - a),$$

where "a" and "b" are the bounds of the support of A. Refer to Fig. 4 illustrating the character of these two requirements along with their conflicting nature: the fuzzy set in Fig. 4(a) is very "specific" yet it does carry a very limited experimental evidence (note a limited number of data "embraced" by the fuzzy set). Fig. 4(b) reveals an opposite situation: we have an information granule of a large size (not being specific) but supported by a significant number of data points.

We can combine these two in a form of a single index Q being a ratio of these two

$$Q \sum_{k=1}^{N} A(x_k)/\text{measure}(\text{supp}(A)) = \sum_{k=1}^{N} A(x_k)/(b - a). \tag{1}$$

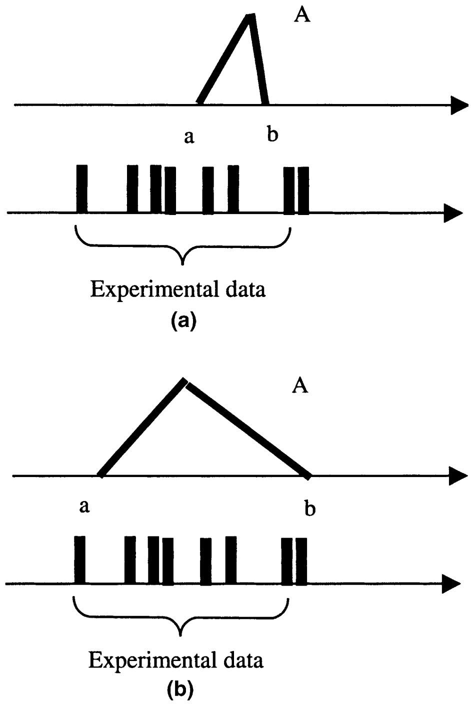


Fig. 4. Information granules-fuzzy sets satisfying one of the optimization criteria; see a detailed description in the text.

Apparently, in light of the above requirements, Q has to be maximized

$$\max_{\mathbf{p}} Q$$

with **p** denoting a collection of the parameters of the membership function to be optimized.

The detailed way in which this optimization is carried out will be discussed in the subsequent section.

5. The design of information granules: a detailed algorithm

In what follows, we confine ourselves to modal fuzzy sets. This helps us design a general algorithm while not loosing generality of the resulting construct. We can envision that the property of modality is a highly desired property retaining the semantics of the information granule. Moreover the modal nature of the membership function helps us handle the development of the fuzzy set by looking into its decreasing portion and increasing portion separately.

The process of the formation of the information granules can be split into two phases:

- Determining the numeric representative of the data set.
- Building the detailed membership function (here we assume that its form is given a priori).

In the first phase, we may consider the numeric representative of the data to be the first, quite rough descriptor of **X**. This phase does not involve granular entities at all but alludes to a numeric "compression" of the data set. In this case numerous, well-known methods exist: a mean value, median, etc. Anticipating the second, more refined phase, our choice is to proceed with the median. Median is a robust estimator so its value does not depend on any outliers (the property that does not hold for the mean value). The calculations of the median are also straightforward. If **X** is ordered, the median splits the data set in halves. If **X** is unordered, the median (med) is a solution to the following L_1 - optimization problem

$$\min_m \sum_{k=1}^{N} |x_k - m| = \sum_{k=1}^{N} |x_k - \mathrm{med}|.$$

The median is taken as the modal value of the fuzzy set. This splits the data into two subsets that are processed separately leading to the computations of the left- and right-hand portion of the membership function of A. The parameters of A are then determined separately. In some sense, we can view the second phase of the formation of the information granules as a refinement of the "compression" scheme already initiated by the numeric representative of **X** (that is its median). Here, two main approaches can be exercised, Fig. 5:

- *Data-driven*: we select the values of the parameters of the membership function based on the finite number of data meaning that they assume some discrete values implied by the original data.
- Optimization approach.

The first method is straightforward and does not require any excessive optimization effort. We sweep through all data points considering each of them to

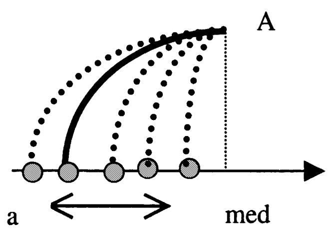


Fig. 5. Detailed computing of the parameter (cutoff point a) of the fuzzy set.

be a potential value of the parameter of the membership function (cutoff point, that is a). The one that maximizes the performance index

$$Q(a) = \sum_{\substack{k=1 \\ x_k<\text{med}}}^{N} A(x_k)/(\text{med} - a)$$

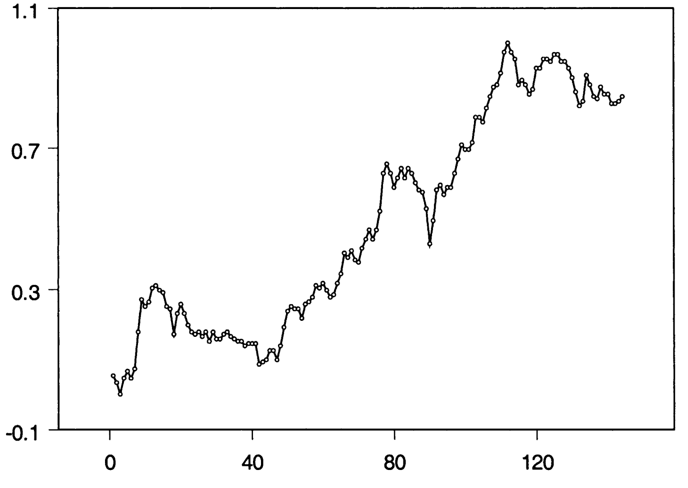


Fig. 6. A synthetic time series.

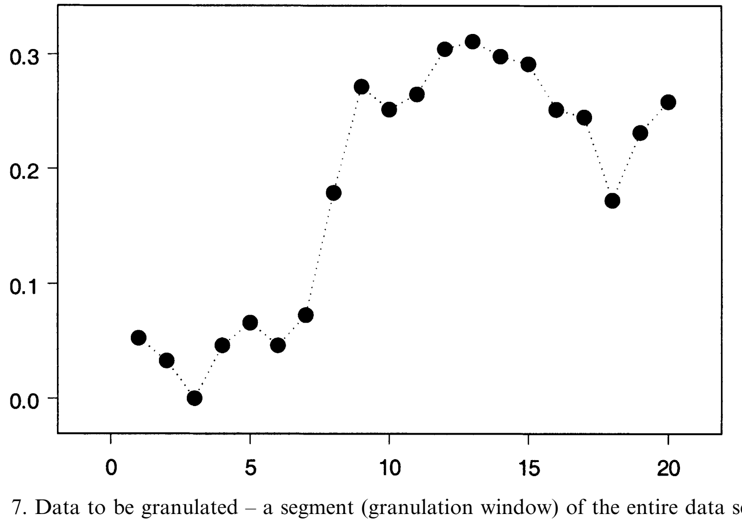


Fig. 7. Data to be granulated – a segment (granulation window) of the entire data set.

forms the solution to the problem $Q(a_{\mathrm{opt}}) = \max Q(a)$. Note that in the above formulation we were dealing with the increasing portion of the membership function. Evidently, the same process is carried out for the decreasing portion of the fuzzy set.

The second approach, that is a full-fledged optimization method, maximizes Q over all possible values of "a". This process could lead to the higher values of the performance index yet it comes with more profound computational overhead.

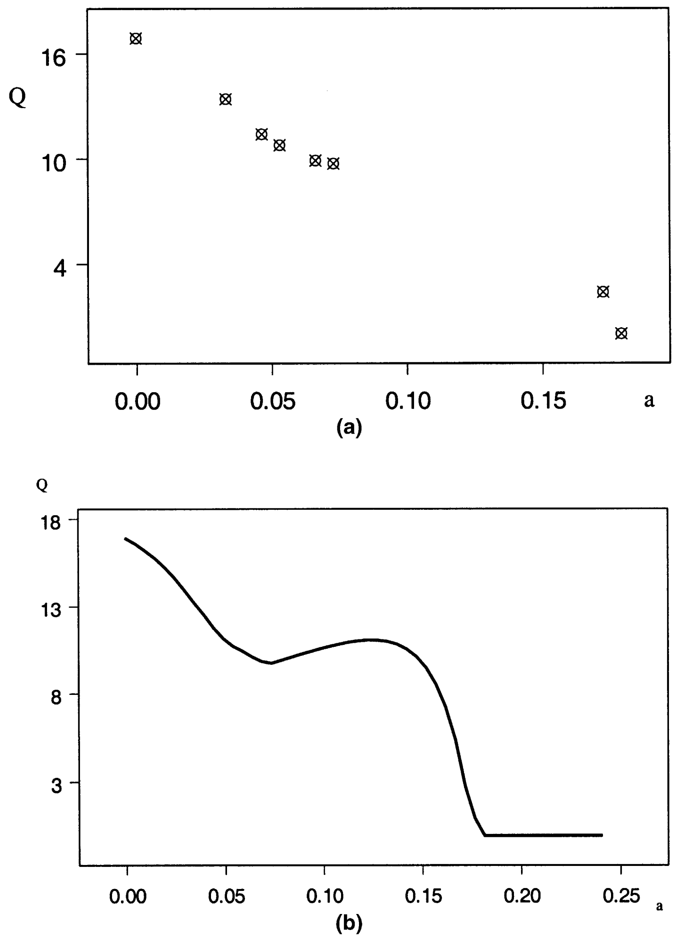


Fig. 8. Performance index Q versus the lower bound of the fuzzy set: direct enumeration (a) and optimization (b).

As a numeric illustration, we discuss a synthetic data set is shown in Fig. 6 which represents a discrete time series.

To illustrate the underlying optimization process, let us consider a section of 20 successive samples (granulation window) of the synthetic time series, Fig. 7. The granulation is realized with the use of the parabolic fuzzy set (parabolic membership function).

The computed median of this granulation window is equal to 0.245033.

Determining the bounds of the parabolic membership function, we get the following values of the cutoff points:

- Using the direct method these are equal to 0.00 and 0.245, respectively. Note that the range of the amplitude of this segment of the time series is equal to [0.000000, 0.311258].

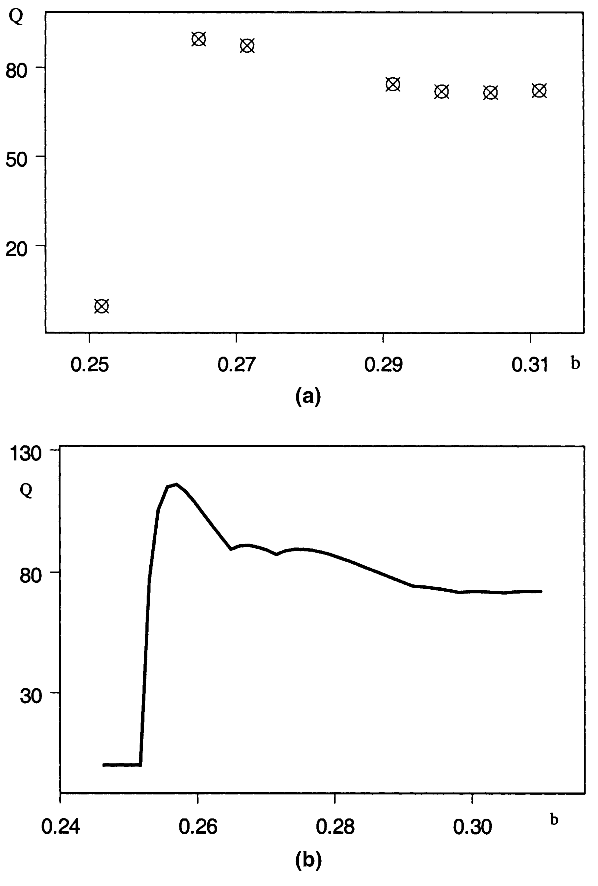


Fig. 9. Performance index Q versus the upper bound of the fuzzy set: direct enumeration (a) and optimization (b).

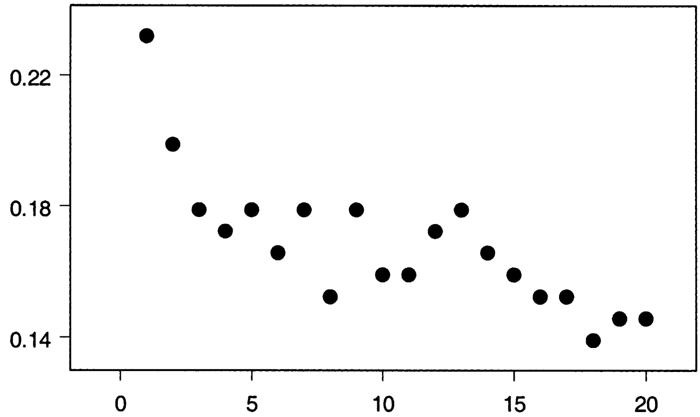


Fig. 10. Data to be granulated.

- Optimizing the cutoff points, we obtain the values equal to 0.00 and 0.257. These are different and produce slightly higher values of the performance index Q.

The plots of the performance index obtained for the two methods are illustrated in Figs. 8 and 9.

In the sequel, we consider another granulation window coming from the entire data set and illustrated in Fig. 10.

This segment of data is spread between 0. 139 and 0.232 with the median equal to 0.1655. The results of computing the bounds of the parabolic fuzzy set are contained in Figs. 11 and 12.

6. Some applications of granular models of signals

Signal processing is predominantly numeric. Numeric data are processed in a linear or nonlinear fashion. Once we get into information granules, they give rise to a new dimension of signal analysis and signal processing. Interestingly, some of already existing pursuits of signal analysis and classification such as syntactic pattern recognition dwell on symbolic elements, see [15–17]. In this section, we elaborate on two interesting ideas that directly originate from the ideas of granular computing.

6.1. Predictive description of granular models

The first-order linear systems are in common usage. We expand on it by proposing a first-order granular dynamic model linking the actual information granule with the predicted one. The underlying formula reads as follows:

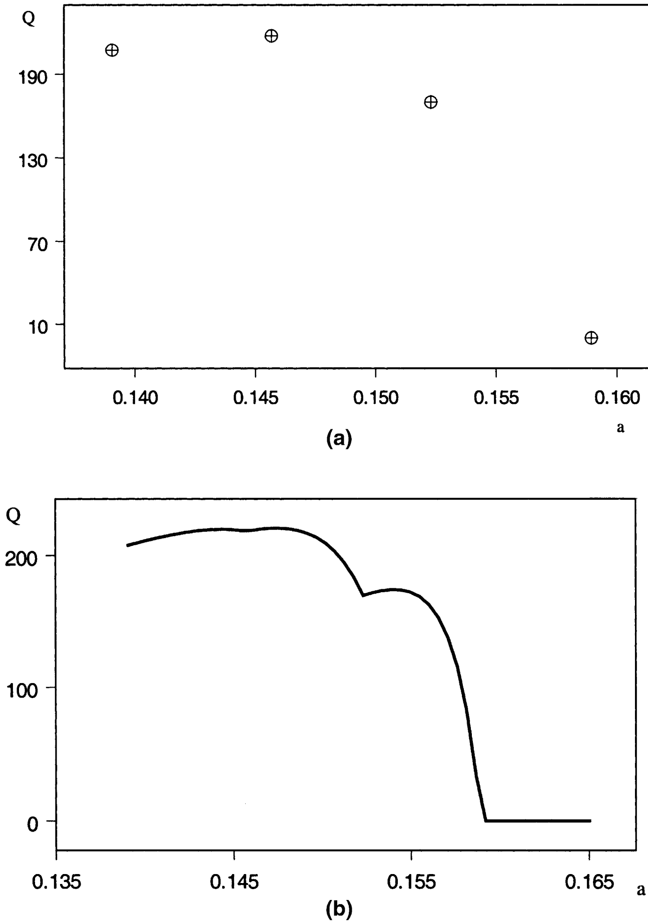


Fig. 11. Performance index Q versus the lower bound of the fuzzy set: direct enumeration (a) and optimization (b).

$$B = A \oplus \partial A\, T,$$

where A and ∂A are the information granules of the signal describing its current status (namely, an amplitude A and the trend of the granule described by the first-order derivative, ∂A). Both A and ∂A are determined as fuzzy sets (triangular, parabolic, etc.) based on the numeric data contained in the granulation windows. More specifically, A is obtained through the direct enumeration of the data $\{x_1, x_2, \ldots, x_N\}$ whereas ∂A originates from the same construction applied to $\{x_2 - x_1, x_3 - x_2, \ldots, x_N - x_{N-1}\}$. The size of the temporal granule is

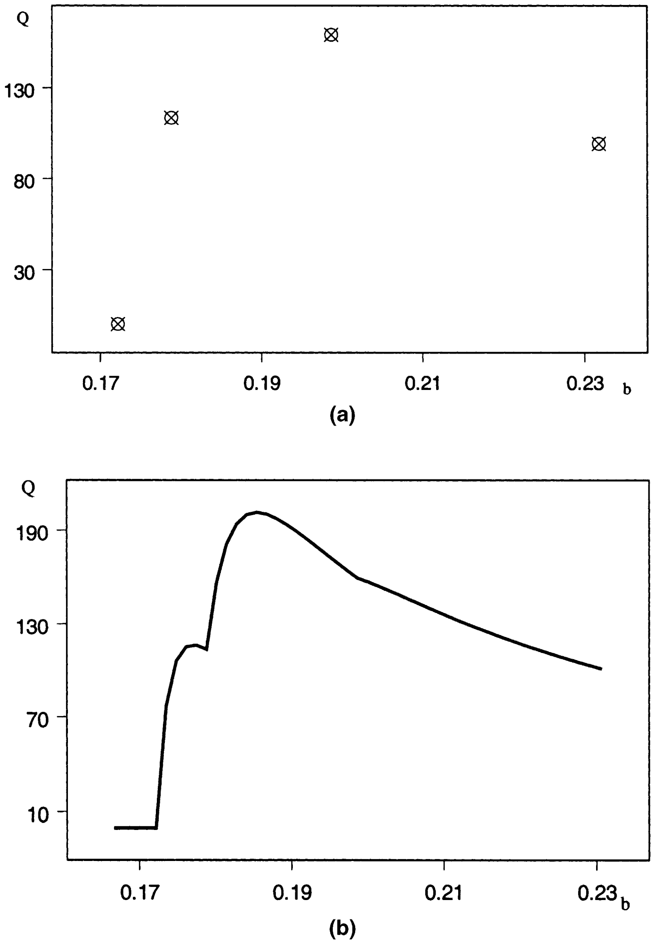


Fig. 12. Performance index Q versus the upper bound of the fuzzy set: direct enumeration (a) and optimization (b).

denoted by T. The above expression is a shorthand symbolic notation and requires some clarification: First, the operation of addition need to be treated in a sense of addition of two fuzzy numbers (with given membership functions). Second, the multiplication occurring above is completed for a single numeric value (T), the result is easy to compute and this multiplication does not affect the form (class) of the membership function. As a matter of fact, it realizes a simple scaling process. The size of the temporal granule (T) modulates a level of impact of the changes (∂A) on the predicted information granule and is subject to some optimization procedure. In other words, we look for an optimal value of T, T_{opt}, so that the predicted information granule B matches the information granule B' manifesting in the time series.

6.2. Condensation of numeric signals

Naturally, information granules help "condense" the signal and represent it in the form of the sequence of information granules – fuzzy numbers. In a nutshell, this type of condensation moves us up from the numeric level up to the symbolic processing layer. The size of the granules (granulation windows and subsequently fuzzy sets) implies a level of abstraction that is achieved. The level of abstraction is essential in many possible ways. First, we can develop models that capture and articulate relationships at the higher level of abstraction. This leads to models that are easier to understand and which give a

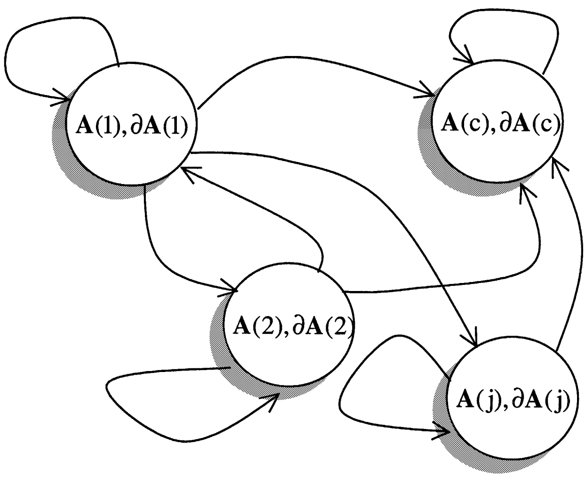


Fig. 13. The web of prototypical information granules.

Table 1
Information granules (the size of the segments – granulation windows is equal to 20 elements)

$A(K)$			$\partial A(K)$		
0.000000	0.245033	0.264901	−0.039735	0.006623	0.039735
0.145695	0.165563	0.198675	−0.026490	−0.006623	0.013245
0.086093	0.218543	0.278146	−0.006623	0.006623	0.013245
0.384106	0.417219	0.523179	−0.026490	0.026490	0.059603
0.569536	0.602649	0.668874	−0.046358	0.000000	0.066225
0.847682	0.880795	1.000000	−0.026490	0.013245	0.039735
0.821192	0.900662	0.966887	−0.039735	−0.006622	0.013245

better insight into the nature of the phenomenon. The information granules serve as basic building blocks used afterwards in a variety of models. For instance, syntactic pattern recognition dwells on a family of structural elements, see [4,8–10,15]. In this case these structural elements are just fuzzy numbers. For each granulation window, we define a fuzzy set capturing the amplitude of the signal and another fuzzy set describing its changes. This leads to the pair $(A(K), \partial A(K))$ (in contrast to the previous notation in the original space, we use a capital letter to denote that this concerns a different time scale). More

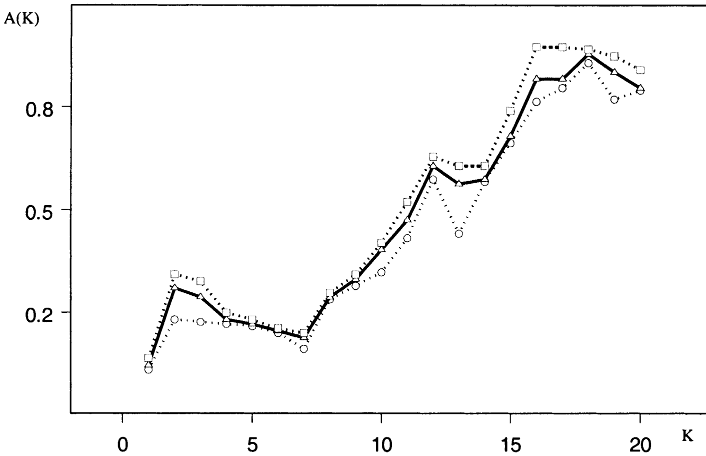


Fig. 14. Plots of $A(K)$ (granulation window equal to 7).

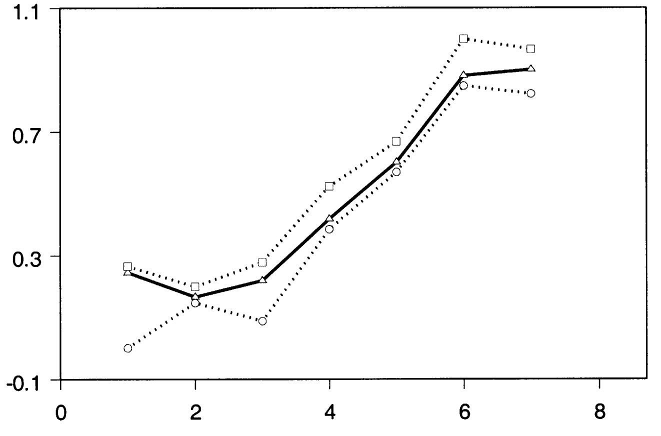


Fig. 15. Plots of $A(K)$ (granulation window equal to 20).

descriptively, the granulation process and the ensuing representation can be described as follows:

$$\{x(1), x(2), \ldots, x(k), \ldots\} \;\Rightarrow\; \{(A(1), \partial A(1)), \ldots (A(K), \partial A(k)), \ldots\}.$$

While the above representation gives us a certain insight into the sequence of the information granules, they can be connected together in the form of a web

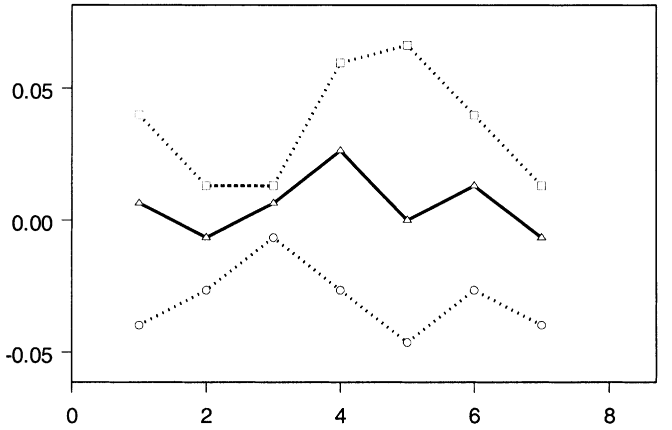


Fig. 16. Plots of $\partial A(K)$ (granulation window equal to 20).

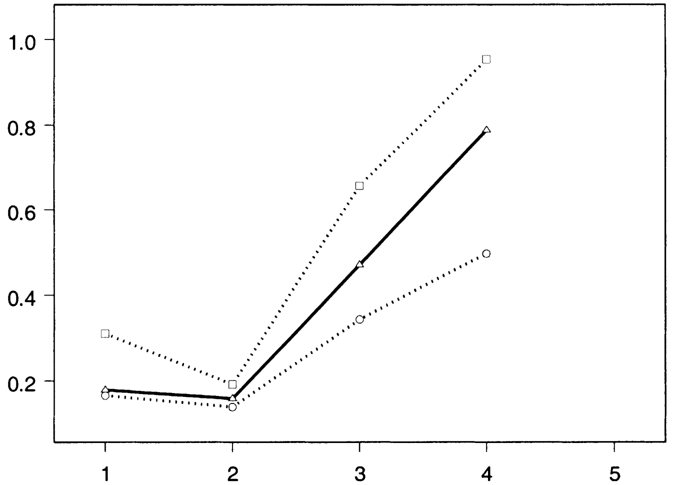


Fig. 17. Plots of $A(K)$ (granulation window equal to 30).

of generic entities. More specifically, as the number of the information granules (or their combinations) $A(K)$, $\partial A(K)$ could be high, they are clustered (grouped) and then used as the components of the web. Denote the prototypes (representatives) of the clusters by $(\mathbf{A}(1), \partial\mathbf{A}(1)), \ldots, (\mathbf{A}(c), \partial\mathbf{A}(c))$ where "c" stands for the number of the clusters. The clustering method is secondary to this problem; a FCM method or its relative could be a plausible choice [1]. The collection of the information granules is then mapped onto the structure of the prototypes. This mapping is realized by determining the connections between the nodes (prototypes), Fig. 13.

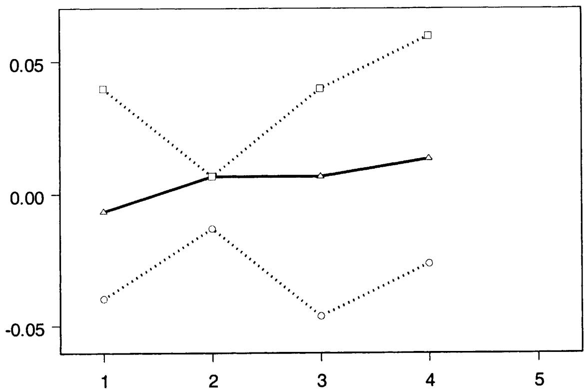


Fig. 18. Plots of $\partial A(K)$ (granulation window equal to 30).

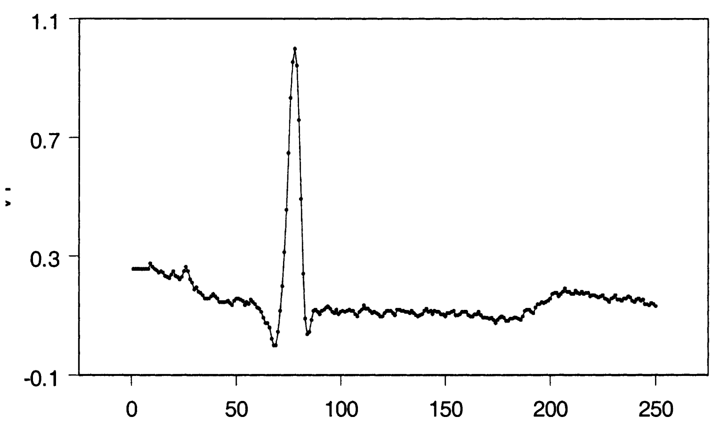


Fig. 19. An original ECG signal (QRS complex).

6.3. Experimental studies

In this section, we continue the numerical analysis we started in Section 5. Proceeding with the signal in Fig. 6, we get its description summarized in Table 1. The table includes information granules describing the amplitude of the signal as well as its changes, ∂A.

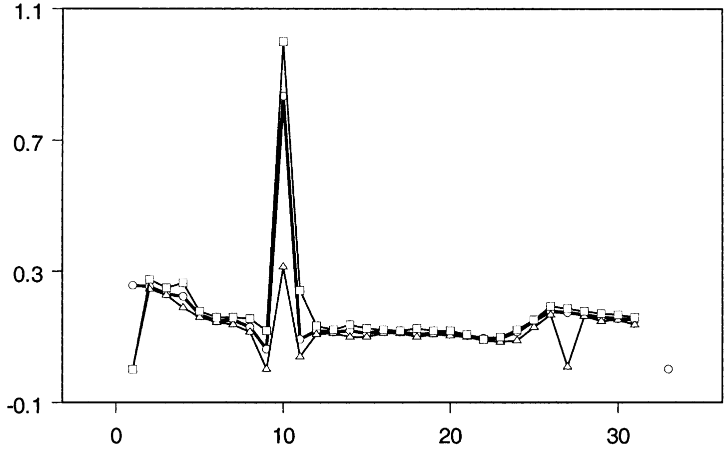


Fig. 20. Condensed ECG signal (the size of the granulation window equal to 8).

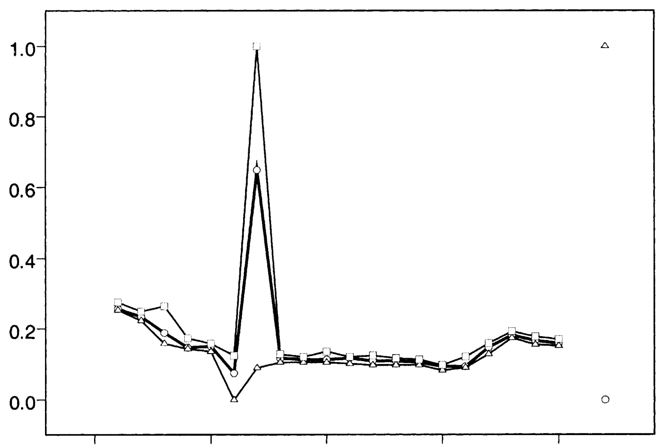


Fig. 21. Condensed ECG signal (the size of the granulation window equal to12).

The series of figures, Figs. 14–18 illustrates the plots of information granules ($A(K)$ and $A(K)$) for several selected values of K. The bounds of the parabolic fuzzy sets are marked using a dotted line. An observation is in place: the larger the granulation window, the more synthetic and concise the description becomes while the granules themselves get broader.

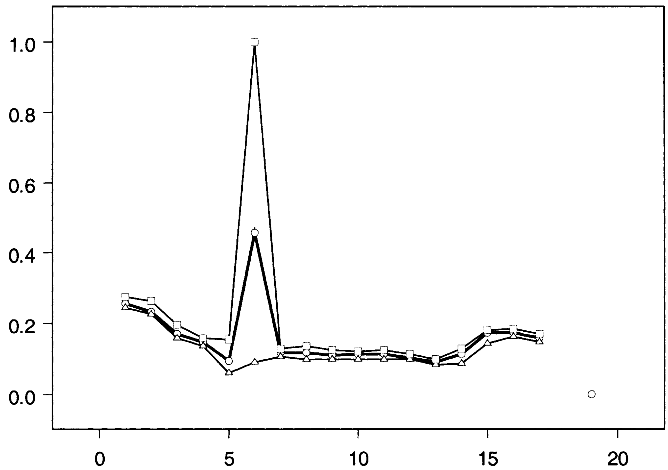


Fig. 22. Condensed ECG signal (the size of the granulation window is equal to 14).

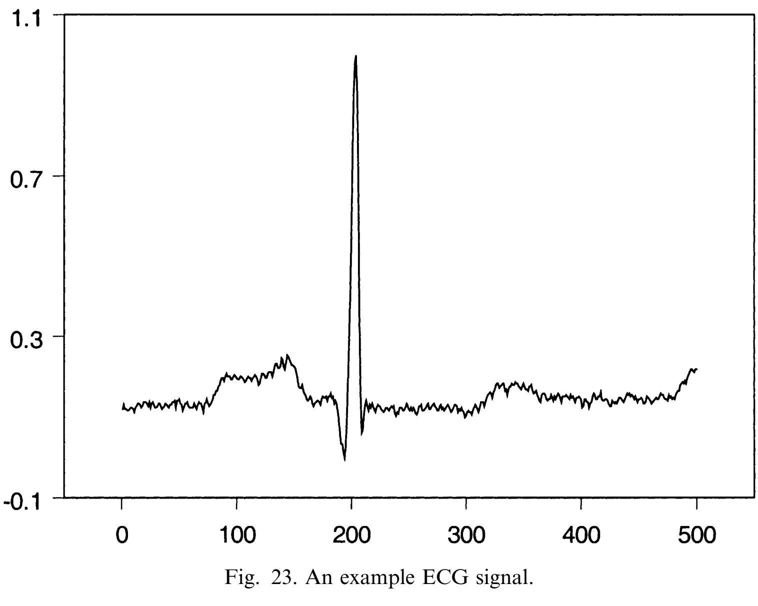
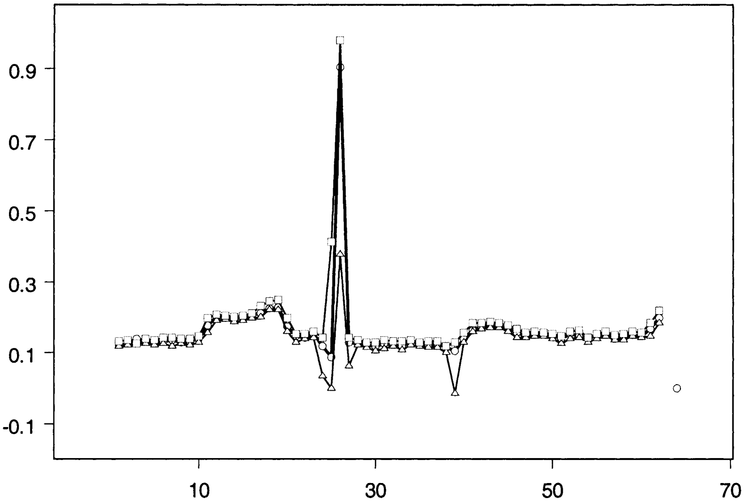


Fig. 23. An example ECG signal.



(b)

Fig. 24. Granulation of the ECG signal; segmentation window is equal to 8.

In the current example, we consider an ECG signal, Fig. 19 and proceed with its granulation. By varying the size of the granulation window, various granular representations of the same initial signal are obtained, see Figs. 20–22.

Fig. 23 illustrates another ECG signal while Fig. 24 shows the results of its temporal granulation.

In the sequel, we exploit the predictive capabilities of the model. We consider two temporal windows of 8 and 9 successive time samples. The performance index (V) expressed as a sum of absolute distances between the parameters of the predicted information granule B and the one (B') resulting from the granulation of the experimental data,

$$V = \sum_K \{|a(K) - a'(K)| + |m(K) - m'(K)| + |b(K) - b'(K)|\}.$$

In the above performance index, $a(K)$, $m(K)$, $b(K)$ and $a'(K)$, $m'(K)$, $b'(K)$ are the parameters of B and B', respectively. In both cases, refer to Fig. 25, V exhibits a clearly visible minimum. Its location depends upon the size of the information granules used in the model.

7. Conclusions

In this study, we have discussed the role of information granulation and the ensuing information granules in description of time series. The detailed

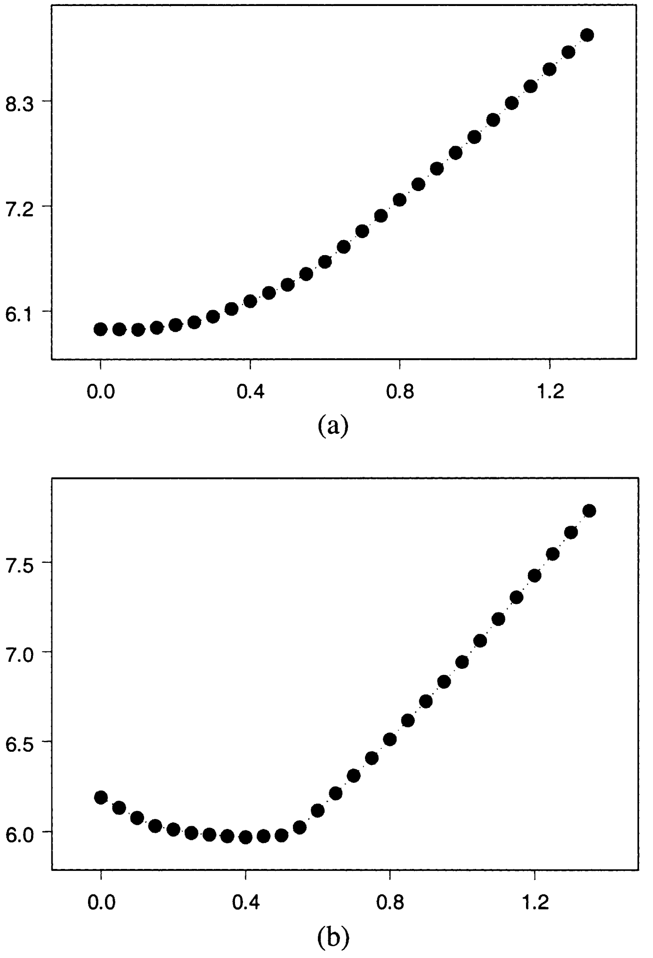


Fig. 25. V versus T for two selected values of the granulation window equal to 8 (a) and 9 (b).

algorithm of information granulation produces information granules – fuzzy sets that are legitimate in terms of experimental data while still sustain their specificity. It has been shown that information granules can be are regarded as generic conceptual entities contributing to the description of numeric time series. In this capacity, they are used as building blocks aimed at achieving high level, compact, and comprehensible models of signals. More importantly, the phase of information granulation could be viewed as a prerequisite to more synthetic and abstract processing such as the one encountered in syntactic pattern recognition.

Acknowledgements

The support from the Natural Sciences and Engineering Research Council of Canada (NSERC) is gratefully acknowledged.

References

[1] J.C. Bezdek, Pattern Recognition with Fuzzy Objective Function Algorithms, Plenum Press, New York, 1981.
[2] K. Cios, W. Pedrycz, R. Swiniarski, Data Mining Techniques, Kluwer Academic Publishers, Boston, 1998.
[3] B.S. Everitt, Cluster Analysis, Heinemann, Berlin, 1974.
[4] S.L. Horowitz, A syntactic algorithm for peak detection in waveforms with applications to cardiography, Communications of the ACM 18 (5) (1975) 281–285.
[5] F. Höppner, F. Klawonn, R. Kruse, T. Runkler, Fuzzy Cluster Analysis, Wiley, Chicester, 1999.
[6] A. Kandel, Fuzzy Mathematical Techniques with Applications, Addison-Wesley, Reading, MA, 1986.
[7] N. Kasabov, Foundations of Neural Networks, Fuzzy Systems, and Knowledge Engineering, MIT Press, Cambridge, MA, 1996.
[8] M. Kundu, M. Nasipuri, D.K. Basu, Knowledge-based ECG interpretation: a critical review, Pattern Recognition 33 (2000) 351–373.
[9] G. Papakonstantinou, E. Skordolakis, F. Grtazali, A attribute grammar for QRS detection, Pattern Recognition 19 (4) (1986) 297–303.
[10] G. Papakonstantinou, An interpreter of attribute grammars and its application to waveform analysis, IEEE Trans. Software Eng. SE-7 (3) (1981) 279–283.
[11] Z. Pawlak, Rough Sets: Theoretical Aspects of Reasoning About Data, Kluwer Academic Publishers, Dordrecht, 1991.
[12] W. Pedrycz, Computational Intelligence: An Introduction, CRC Press, Boca Raton, FL, 1997.
[13] W. Pedrycz, F. Gomide, An Introduction to Fuzzy Sets, Cambridge, MIT Press, Cambridge, MA, 1998.
[14] W. Pedrycz, Fuzzy equalization in the construction of fuzzy sets, Fuzzy Sets and Systems 119 (2001) 329–335.
[15] E. Skordolakis, Syntactic ECG pattern processing: a review, Pattern Recognition 19 (4) (1986) 305–313.
[16] P. Trahanias, E. Skordolakis, Syntactic pattern recognition of the ECG, IEEE Trans. Pattern Anal. Mach. Intell. PAMI-12 (7) (1990) 648–657.
[17] K. Udupa, I.S.N. Murphy, Syntactic approach to ECG rhythm analysis, IEEE Trans. Biomed. Eng. BME-27 (7) (1980).
[18] L. A Zadeh, Fuzzy sets and information granularity, in: M.M. Gupta, R.K. Ragade, R.R. Yager (Eds.), Advances in Fuzzy Set Theory and Applications, North-Holland, Amsterdam, 1979, pp. 3–18.
[19] L.A. Zadeh, Fuzzy logic = computing with words, IEEE Trans. Fuzzy Syst. 4 (2) (1996) 103–111.
[20] L.A. Zadeh, Toward a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic, Fuzzy Sets and Systems 90 (1997) 111–117.