Reichenbach's Common Cause Principle

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Reichenbach's Common Cause Principle:

compact formulation:

No correlation without causation

Explicitly:

If two events A, B are probabilistically correlated, then either there is a causal connection between A and B that is responsible for the correlation or there is a third event C, a (Reichenbachian) common cause which brings about the correlation.

What is Reichenbach's Common Cause Principle?

- A law of Nature?
- A metaphysical claim about the causal structure of the World?
- A methodological principle guiding scientific research?

Main message of talk:

(Aggressive formulation)

If falsifiability is taken as necessary condition for a claim to be a law of Nature then Reichenbach's Common Cause Principle is not a law because it is not falsifiable

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If falsifiability is taken as necessary condition for a claim to be a law of Nature then Reichenbach's Common Cause Principle is not a law because it is not falsifiable

(Gentle formulation)

It is more difficult to falsify the Common Cause Principle than one may think

Structure of talk:

- Reichenbach's notion of common cause
- Local common cause completeability (notion+proposition)
- Common cause completeness (notion+propositions)
- Comments on
  - common cause completeability of quantum probability spaces
  - Common common causes

(blue = on request/if time permits)

Definition:
Reichenbach's notion of common cause

$(\mathcal{S}, p)$ classical probability space

$C \in \mathcal{S}$ is a common cause of the correlation

$$p(A \cap B) > p(A)p(B)$$

if

$$\begin{aligned}
p(A \cap B|C) &= p(A|C)p(B|C) \\
p(A \cap B|C^{\perp}) &= p(A|C^{\perp})p(B|C^{\perp}) \\
p(A|C) &> p(A|C^{\perp}) \\
p(B|C) &> p(B|C^{\perp})
\end{aligned}$$

Definition:

The probability space $(\mathcal{S}, p)$ is called

common cause incomplete

if it contains a pair of events A, B that are correlated with respect to p but there is no common cause in $\mathcal{S}$ of the correlation

common cause complete (closed)

if it contains a common cause of every correlation it predicts

- There exist trivially common cause complete probability spaces

$$\|$$

containing no correlations at all – not interesting

- There exist common cause incomplete probability spaces

Common cause incomplete probability spaces are a threat for the Common Cause Principle.

Can this threat be met?

$$\Downarrow$$

Can a common cause incomplete probability space be extended in such a way that the extension contains a common cause of the correlation?

Definition:

The probability space $(\mathcal{S}', p')$ is called an extension of $(\mathcal{S}, p)$ if there exists a Boolean algebra embedding h of $\mathcal{S}$ into $\mathcal{S}'$ such that

$$p(X) = p'(h(X)) \qquad \text{for all } X \in \mathcal{S}$$

The embedding homomorphism h takes each correlation in $(\mathcal{S}, p)$ without distortion into a correlation in $(\mathcal{S}', p')$ $\Rightarrow$ it does make sense to talk about the common cause in $\mathcal{S}'$ of a correlation in $(\mathcal{S}, p)$

Definition:

$(\mathcal{S}, p)$ is called common cause completeable with respect to a correlated pair A, B (also called **locally** common cause completeable) if there exists an extension $(\mathcal{S}', p')$ of $(\mathcal{S}, p)$ such that the extension contains a common cause of the correlation between A and B

Theorem:

Every probability space $(\mathcal{S}, p)$ is common cause completeable with respect to any pair (hence with respect to any finite set) of correlated events A, B in $\mathcal{S}$

Significance of Theorem:

It is always possible to defend Reichenbach's Common Cause Principle against attempts of falsification by referring to

hidden common causes

$\|$

not part of the original, common cause incomplete event structure

We could end here!

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But:

A philosopher very insistent on showing that Reichenbach's Common Cause Principle is falsifiable could say this:

Local common cause completeability

does not imply:

There exists a common cause complete extension $(\mathcal{S}', p')$ of a common cause incomplete $(\mathcal{S}, p)$

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Problem:

Do common cause complete probability spaces exist?

Definition:

$(\mathcal{S}, p)$ is an atomless probability space if
for A with $p(A) > 0$
there exists $0 \neq B \subset A$
with $0 \neq p(B) < p(A)$

Example: $([0, 1], p)$ $p =$ Lebesgue measure

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Atomless probability spaces are common cause closed

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Insistent philosopher:

Not surprising that atomless probability spaces are common cause closed: they are very large :

- have a continuum number of random events
  hence
- highly non-constructive
  hence
- are empirically very inaccessible

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Insistent philosopher:

Problem

Can finite probability spaces be common cause complete?

Theorem:

If the Boolean algebra $\mathcal{S}_n$ has a finite number of elements then $(\mathcal{S}_n, p)$ is not non-trivially common cause complete

A finite probability space contains more correlations than it can explain with the help of common causes

Still:

Insistent Philosopher cannot claim victory:

Reichenbach's Common Cause Principle:

$$\Updownarrow$$

Common causes only for correlations between

causally independent $R_{ind}(A,B)$ events

$$\Downarrow$$

The definition of common cause closedness is unreasonably strong:

it leaves no room for causal dependence.

Definition:

$(\mathcal{S}, p)$ is causally closed (complete)
with respect to a causal independence relation R_{ind} on $\mathcal{S}$
if $\mathcal{S}$ contains a common cause of every correlation between elements
A, B such that $R_{ind}(A, B)$ holds

Demolishing

Insistent Philosopher:

By making every probability space $(\mathcal{S}, p)$

causally complete

by defining:

$R_{ind}(A, B)$ holds

whenever A and B are correlated

but there is no common cause of this correlation in $\mathcal{S}$

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Too cheap!

We need a disciplined definition of causal independence!

Intuition: causal independence of A and B should imply that from the presence or absence of A one should not be able to infer either the occurrence or non-occurrence of B, and conversely: presence or absence of B should not entail occurrence or non-occurrence of A.

Definition:

$A, B \in \mathcal{S}$ are called logically independent if

$$\begin{array}{ccc} A \not\subseteq B, \quad A^{\perp} \not\subseteq B & , & A \not\subseteq B^{\perp}, \quad A^{\perp} \not\subseteq B^{\perp} \\ B \not\subseteq A, \quad B^{\perp} \not\subseteq A & , & B \not\subseteq A^{\perp}, \quad B^{\perp} \not\subseteq A^{\perp} \end{array}$$

Definition:

Two Boolean subalgebras $\mathcal{L}_1, \mathcal{L}_2$ of $\mathcal{S}$ are called

logically independent if

any $0 \neq A \in \mathcal{L}_1$ and $0 \neq B \in \mathcal{L}_2$ are logically independent

i.e. if

$$A \cap B \neq 0$$

for $0 \neq A \in \mathcal{L}_1 \qquad 0 \neq B \in \mathcal{L}_2$

Definition:

$(\mathcal{S}, p)$ is causally closed with respect to logically independent sub Boolean lattices $\mathcal{L}_1, \mathcal{L}_2$ if $\mathcal{S}$ contains a common cause of every correlation between $A \in \mathcal{L}_1$ and $B \in \mathcal{L}_2$

Theorem:

$(\mathcal{S}_5, p_u)$ is non-trivially causally closed with respect to every pair of logically independent Boolean subalgebras

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$\uparrow$

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Very strong causal completeness !!

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Surprising!

Very strong causal completeness !!

The very strong causal completeness of $(\mathcal{S}_5, p_u)$ is truly exceptional:

Theorem:

If $(\mathcal{S}_n, p)$ is not $(\mathcal{S}_5, p_u)$ then $(\mathcal{S}_n, p)$ is not non-trivially causally closed with respect to every pair of logically independent Boolean subalgebras

Theorem:

For any $n \geq 5$, if $\mathcal{S}_n$ is a finite Boolean algebra generated by n atoms, then there exists a probability measure p on $\mathcal{S}_n$ and there exist two logically independent Boolean subalgebras $\mathcal{L}_1, \mathcal{L}_2$ of $\mathcal{S}_n$ such that $(\mathcal{S}_n, p)$ is causally closed with respect to $(\mathcal{L}_1, \mathcal{L}_2)$.

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- It is not known how typical or untypical common cause completeness is (with respect to an R_{ind} stronger than logical independence) in finite probability spaces
- There is no straightforward test to tell if a probability space is causally complete

Summary of technical claims:

- Atomless probability spaces are common cause closed
- Finite probability theories may or may not be causally closed with respect a causal independence relation stronger than logical independence
- Common cause incomplete classical probability spaces (finite or not) are always common cause completeable with respect to a fixed, finite set of correlations
- Common cause incomplete typical non-commutative probability spaces are common cause completeable with respect to all correlations in a fixed state

Summary of philosophical points:

- One can always defend Reichenbach's Common Cause Principle by referring to "hidden" common causes
- One cannot falsify Reichenbach's Common Cause Principle by claiming that (reasonably defined) causally closed probabilistic theories are impossible (mathematically)
- To falsify Reichenbach's Common Cause Principle one has to require further conditions on the common causes beyond those in Reichenbach's definition of common cause
(e.g. locality)

At a minimum one can safely claim:

Falsifying Reichenbach's Common Cause Principle is indeed more difficult than one may have thought